• Show all your work and write complete and coherent answers.

• You must show your ISU student ID card or driver’s license if asked.

• You should bring to the exam a calculator capable of taking square roots. You may not use your cellphone as a calculator. You may not share calculators.

• Provide units for numerical answers for quantities that have units (e.g., dollars or thousands of dollars).

• Show all of the steps that you used to get your final answer. If you do not show your work you will not get full credit and no partial credit can be given in the case of incorrect answers.

• If you show clearly the steps needed to arrive at a final numerical answer, you will receive full credit even if the final calculation is not complete. For example \( \Pr(Z \geq z) = 1 - 0.95 \) is just as good as \( \Pr(Z \geq z) = 1 - 0.95 = 0.05 \).

• We strongly recommend the use of a pencil (with an eraser) rather than a pen.

• Please write as clearly and neatly as possible. If I cannot read your answers, I cannot give you any credit.

• Feel free to ask for more paper if you need more space.

Take the time to carefully read all the questions on the exam! GOOD LUCK !!!

Form A

I have neither given nor received any unauthorized aid in completing this exam

Signed ______________________________

Total score:
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1. **TRUE or FALSE:** Identify which of the following are valid statements and which ones are false.

(A)  **T F** A p-value larger than the level of significance \( \alpha \) indicates sufficient evidence to prove that the null hypothesis is true.

(B)  **T F** A defendant in a criminal trial is considered not guilty (i.e. innocent) until proven guilty. If there is insufficient evidence to establish guilt, a jury must consider a defendant to be not guilty even when it is true that the defendant committed the crime.

   The same analogy holds true in hypothesis testing where the statement in the null hypothesis corresponds to considering the defendant not guilty. Evidence against the null hypothesis is established based on the available data. When there is little evidence then the null hypothesis cannot be rejected and we continue to believe in it.

(C)  **T F** A \( t \)–statistic counts the number of estimated standard errors a sample mean is above or below the hypothesized value \( \mu_0 \).

(D)  **T F** A 95% confidence interval for the average distance Americans walk every day ranges from 2.14 to 5.85 miles. We can interpret this confidence interval as follows: “95% of all American walk on average between 2.14 to 5.85 miles every day.”

(E)  **T F** Every normal distribution is bell-shaped but not every bell-shaped distribution is a normal distribution.

(F)  **T F** The \( t \)--distribution has a mean of 0.

2. **Multiple Choice.** Choose the best response(s) and circle your answer(s) clearly.

(A)  **(Circle one.)** An alternative hypothesis can also be called:
   i. a contradictory hypothesis
   ii. an agreeing hypothesis
   iii. a null hypothesis
   iv. a superior hypothesis

(B)  **Circle all that apply.** Averaging observations from a random sample
   i. reduces variation
   ii. increases variation
   iii. does not change the variation
   iv. allows the use of bell-shaped distributions provided the sample size is sufficiently large
   v. eliminates bias
   vi. increases precision
   vii. reduces precision
   viii. none of the above
(C) **(Circle one.)** Assume I want to build two 95% confidence intervals using two independent, random samples of size \( n = 50 \), where the population standard deviation is known to be 4. The confidence interval 1 has a sample mean of 5 and confidence interval 2 has a sample mean of 8. Which interval is wider?
   i. Confidence Interval 1
   ii. Confidence Interval 2
   iii. Both have the same width.
   iv. We cannot know without additional information.

(D) **Fill in the blank such that the sentence is statistically correct and statistically meaningful.** While more than one correct answer may be possible some answers are better than others.
   i. When using data to estimate a population parameter, we can minimize the chance of obtaining a biased estimate by ____________________________.

   ii. The ____________________________ is always the center of every confidence interval.

3. **Chronic Pain.** A study was conducted among individuals with chronic pain to determine how effective acupuncture is in relieving pain. A random sample of 15 subjects were chosen and their sensory rates were measured. Assume that the measurements on sensory rates follow a normal distribution. The mean was calculated to be 8.23 and the standard deviation 1.67.

(A) The degrees of freedom associated with the \( t \)-distribution in this set-up are: \( \text{df=} \) ________

(B) What is the value of \( t_{\frac{\alpha}{2},n-1} \) for the 95% confidence interval for the mean sensory rate? (The value obtained from Table-D, accurate to the nearest 3 decimal places)

\[
t_{\frac{\alpha}{2},n-1} = \underline{\text{_____________}}
\]

(C) What is the margin of error for the 95% confidence interval for the mean sensory rate?
Show work here:

Final Answer: \underline{\text{_____________}}

(D) Calculate the 95% confidence interval for the mean sensory rate.
Show work here:

Final Answer: \underline{\text{_____________}}
4. Wind Power. Windlogics is a company that specializes in providing expert advice and solutions for wind energy. One of the services they provide is forecasts for wind to help determine predictions of power output by wind turbines. One simple quantity they are interested in (to validate their model) is the average wind speed for the day in a particular region (say Story county). For a variety of reasons, it is very difficult to measure the average wind speed for a day across an entire region. For their Story county unit, on October 31, 2015, Windlogics takes a simple random sample of size 400 and calculates a mean wind speed of 16 mph. Based on their expertise, Windlogics knows the standard deviation of wind speed in Story county during the fall is 9 mph.

(A) What is the population parameter of interest in the context of this problem?

(B) Based on this information, provide a 97% confidence interval for the population parameter of interest.

Show work here:

Final Answer: ________________________________

(C) Provide an interpretation for the confidence interval you calculated above.

(D) There are assumptions that are needed to be checked/verified to construct and interpret confidence intervals. State these assumptions. Based on the description provided above, explain whether each of these assumptions is met or not in this situation.

(E) Suppose the true mean wind speed on October 31, 2015 in Story county was 15.35 mph, what is the probability this value is in the interval you constructed above?

The probability is: ____________
(F) (Circle one.) Suppose Windlogics took a sample of 500 instead of 400, what would happen to the width of the confidence interval calculated in (B)?

   INCREASE       DECREASE       NOT ENOUGH INFORMATION

(G) (Circle one.) Now suppose Windlogics took a sample of 500 instead of 400, and increased its confidence level, what would happen to the width of the confidence interval calculated in (B)?

   INCREASE       DECREASE       NOT ENOUGH INFORMATION

5. What does a p-value tell you? Consider the Walleye Trouble question from the current homework 8 assignment. Walleye is a fish that is native to the northern United States and Canada that is known for its excellent taste. As a result it is often overfished in Minnesota lakes, especially Lake Mille Lacs. One way to measure the health of the population of Lake Mille Lacs is through the average length of the fish because fish grow their entire lives. The Department of Natural Resources (DNR) considers the walleye population in a lake to be strong if the mean length of walleye in the lake is more than 17 inches. For this problem assume $\alpha = 0.05$.

Based on a sample of 600 netted fish the DNR tests the following set of hypotheses:

$$H_0 : \mu = 17 \quad \text{versus} \quad H_a : \mu > 17$$

and obtains a test statistic $t = 3$ associated with a p-value of 0.0013.

(A) This p-value can be interpreted as follows:

   i. There is a 0.0013 chance to see a test statistic at least as small as 3 when the mean length of the walleye population is indeed 17 inches.
   ii. There is a 0.0013 chance to see a test statistic at least as small as 3 when the mean length of the walleye population is indeed greater than 17 inches.
   iii. There is a 0.0013 chance to see a test statistic at least as large as 3 when the mean length of the walleye population is indeed 17 inches.
   iv. There is a 0.0013 chance that the mean length of the walleye population is 17 inches.
   v. There is a 0.0013 chance that the mean length of the walleye population is greater than 17 inches.

(B) Based on the size of the p-value the DNR can make the following conclusion:

   i. At the $\alpha = 0.05$ level of significance the DNR has sufficient evidence to conclude that the true mean length of the walleye population is 17 inches.
   ii. At the $\alpha = 0.05$ level of significance the DNR has insufficient evidence to conclude that the true mean length of the walleye population is 17 inches.
   iii. At the $\alpha = 0.05$ level of significance the DNR has sufficient evidence to conclude that the true mean length of the walleye population is greater than 17 inches.
   iv. The DNR has sufficient evidence to conclude that the true mean length of the walleye population definitely is 17 inches.
   v. The DNR has sufficient evidence to conclude that the true mean length of the walleye population definitely is larger than 17 inches.
6. **Supplier.** An auto parts supplier claims to deliver parts to the manufacturer in an average time of 60 minutes or less. The manufacturer suspects that the supplier is not meeting their standard and that the average delivery time is longer. The manufacturer records the time (in minutes) of 24 randomly selected deliveries from this supplier and finds a mean delivery time of 61.79 minutes and standard deviation of 6.58. Assume all delivery times are normally distributed. Does the manufacturer have sufficient evidence to conclude that the supplier is taking longer to deliver than their 60 minute guarantee?

The following analysis is performed in JMP:

(A) State in words what the population mean is (in the context of the problem).

(B) State the null and alternative hypothesis using proper statistical notation.

(C) Calculate the test statistic (round to the nearest 2 decimal places): _______________
   Show your work here:

(D) The associated degrees of freedom for the \( t \)-distribution are: _______________

(E) Based on the information given in the JMP output, identify the \( p \)-value associated with the test statistic above:
   \( p \)-value= _______________

(F) (Circle one.) Based on the above \( p \)-value, can you reject the null hypothesis at the \( \alpha = 0.05 \) level of significance? YES or NO

(G) (Circle one.) Does your decision in (F) indicate that the supplier takes longer than 60 minutes to deliver parts to the manufacturer? YES or NO

-score:
7. **Basketball.** Professional basketball players in the NBA are quite tall compared to the average human. The distribution of NBA basketball player heights follows a normal distribution with mean \( \mu = 79 \) inches and standard deviation \( \sigma = 4 \) inches.

(A) Based on what we learned in class, do you have the tools to answer the following question: “What proportion of NBA players are taller than 82 inches?” If yes, compute proportion/probability, and report the value to 4 decimal places. If no, explain why not.

Show work or provide explanation:

Final Answer: ______________

(B) On average, there are 15 players on an NBA team. What is the distribution of the sample mean player height for an NBA team? Specify exactly the type, mean, and standard error in your answer, and use proper notation.

(C) Based on what we learned in class, do you have the tools to answer the following question: “What proportion of teams of size 15 have a sample mean player height at or above 82 inches?” If yes, compute the proportion/probability, and report the value to 4 decimal places. If no, explain why not.

Show work or provide explanation:

Final Answer: ______________

(D) Regardless of your findings in part (C), assume that you can answer the following question correctly with the tools discussed in class. Consider all possible samples consisting of 15 players. What is the mean height such that 90% of all possible samples of 15 players result in an average/mean equal to or higher than this value. Round your answer to 2 decimal places.

Show work here:

Final Answer: ______________

(E) For which of the above parts (part (A), (B), (C), and (D)) did you have to use the central limit theorem? (Circle all that apply.)

PART (A)  PART (B)  PART (C)  PART (D)  NONE

-score:

HAVE A GREAT WEEKEND!
Table entry for $z$ is the area under the standard normal curve to the left of $z$.

### TABLE A  Standard normal probabilities

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### TABLE A: Standard normal probabilities (continued)

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**Confidence level $C$**

- 50% 60% 70% 80% 90% 95% 98% 99% 99.5% 99.8% 99.9%

**Probability $p$**

- $z^*$
Summary Statistics

\[ \bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i = \frac{y_1 + y_2 + \ldots + y_n}{n} \]

\[ s_y^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2 = \frac{(y_1 - \bar{y})^2 + (y_2 - \bar{y})^2 + \ldots + (y_n - \bar{y})^2}{n-1} \]

\[ s_y = \sqrt{s_y^2} \]

Normal Calculations

\[ X \sim N(\mu, \sigma^2) \quad Z \sim N(0, 1^2) \quad z = \frac{x - \mu}{\sigma} \]

\[ x_p = z_p \cdot \sigma + \mu, \text{ where } p \text{ denotes the } 100 \times p^{th} \text{ percentile.} \]

Sampling Distribution

\[ SE(\bar{X}) = \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}, \quad z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}, \quad \bar{x}_p = z_p \cdot \frac{\sigma}{\sqrt{n}} + \mu, \text{ where } p \text{ denotes the } 100 \times p^{th} \text{ percentile.} \]

\[ \sigma \text{ is known} \]

\[ \bar{x} \pm z_{0.5} \cdot \frac{\sigma}{\sqrt{n}}, \quad m = z_{0.5} \cdot \frac{\sigma}{\sqrt{n}} \]

\[ \sigma \text{ is unknown} \]

\[ \bar{x} \pm t_{\alpha/2, n-1} \cdot \frac{s}{\sqrt{n}}, \quad m = t_{\alpha/2, n-1} \cdot \frac{s}{\sqrt{n}} \]

Hypothesis Testing

\[ t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \]