Useful Equations
✓ Standing Wave : \( y(x,t) = 2A \sin(kx) \sin(\omega t) \)
✓ Harmonics : \( f_n = \frac{nv}{2L} \) (For both ends open or both ends closed/fixed)
  ◦ \( f_n = \frac{n_{\text{odd}}v}{4L} \) (For one end open and one end closed/fixed)
✓ Node : A point on a standing wave which has no oscillatory motion.
✓ Anti-Node : A point on a standing wave with the largest oscillatory motion.
✓ Doppler Shift : \( f_L = \frac{v + v_L}{v + v_S} f_S \) (The positive direction is from the listener (L) to the source (S).)
  ◦ The speed of sound in air at room temperature is 343 m/s

Problems :
1. (*) Shown below is a diagram representing a standing sound wave. Label the pressure nodes and anti-nodes. Label the displacement nodes and anti-nodes.

   ![Standing wave diagram]

   • Each pressure node is a displacement anti-node and vice-versa.

2. (**) What is the fundamental frequency (the lowest frequency harmonic) of an organ pipe 1 meter long? The organ pipe has one closed end and one open end. What is the third harmonic?
   • We know for this case we must use the formula \( f_n = \frac{n_{\text{odd}}v}{4L} \) where \( n_{\text{odd}} = 1 \).
   • The length of the pipe is 1 meter, therefore \( f_n = \frac{n_{\text{odd}}v}{4L} = \frac{(1)(343 \text{ m/s})}{4(1 \text{ m})} = 86 \text{ Hz} \)
   • The third harmonic is \( n_{\text{odd}} = 3 \) : \( f_n = \frac{n_{\text{odd}}v}{4L} = \frac{(3)(343 \text{ m/s})}{4(1 \text{ m})} = 257 \text{ Hz} \)
3. (**) We want to build a guitar. One of the strings must be tuned to an 'A' which has a frequency of 440 Hz. If our guitar string is half a meter long and weighs 60 g, what tension do we need to have in the string to produce the desired harmonic?

- On a guitar both ends are fixed, so \( f_n = \frac{n v}{2L} \) and \( v = \sqrt{\frac{T}{m}} = \sqrt{\frac{TL}{m}} \)

\[
f_n = \frac{n}{2} \sqrt{\frac{T}{mL}}
\]

- \( T = mL \left( \frac{2}{n} f_n \right)^2 \). Clearly the tension can take multiple values, we will simply take it to be \( n = 1 \).

\[
T = (0.06 \text{ kg})(0.5 \text{ m}) \left| 2 \left( 440 \text{ Hz} \right) \right| = 23 \text{ kN} \quad \text{(Clearly a lot of tension)}
\]
- (Clearly a lot of tension) – note that we can reduce the needed tension if we use lighter strings.

4. (**) You are traveling down a highway at 30 m/s when an ambulance blaring a siren at 500 Hz comes up behind you at 35 m/s. What frequency do you hear?

- The direction from listener to source is positive, therefore your velocity is negative, as is the velocity of the ambulance.

\[
f_L = \frac{v - v_L}{v - v_S} f_S = \left( \frac{343 \text{ m/s}}{343 \text{ m/s}} \right) - \left( \frac{30 \text{ m/s}}{35 \text{ m/s}} \right) (500 \text{ Hz}) = 508 \text{ Hz}
\]

5. (**) You are in a car traveling towards a wall and honking your horn at 300 Hz. The sound from your horn reflects off of the wall and back towards you. If the frequency you hear is 500 Hz, how fast are you driving?

- We are moving in the positive direction (because it is from listener to source and we are the listener, the wall can be thought of as the source).

- We can do this in two steps, first determine the frequency 'heard' by the wall, and then treat that frequency as the source of what we hear. The frequency heard by the wall will be \( f_L = \frac{v - v_S}{v - v_L} f_S \) where \( v_S \) is the speed of the car.

- Next, the frequency heard by the car will be \( f'_L = \frac{v + v_S}{v} f_L \) (Because the car is moving and the wall is not). Combining these, \( f'_L = \frac{v + v_S}{v} \left( \frac{v}{v - v_S} f_S \right) = \frac{v + v_S}{v - v_S} f_S \) (In other words, we can treat the wall as a source moving towards us at the same speed as we are moving towards it.

\[
f_L \left( v - v_S \right) = \left( v + v_S \right) f_S \rightarrow v \left( f_L - f_S \right) = v_S \left( f_S + f_L \right) \rightarrow v_S = v \frac{f_L - f_S}{f_S + f_L}
\]

\[
\frac{v_S}{\left( 343 \text{ m/s} \right) \left( 500 \text{ Hz} \right) - \left( 300 \text{ Hz} \right)} \left( 500 \text{ Hz} \right) + \left( 300 \text{ Hz} \right) = 86 \text{ m/s} \left( 192 \text{ mph} \right) \quad \text{(you should probably slow down)}
\]

6. (***) A tornado siren sounds at 900 Hz a distance of 4 km away. If you are standing still, but there is a wind blowing at 15 m/s from the siren towards you, what frequency do you hear?

- 900 Hz – neither the source nor listener is moving so the sound is not Doppler shifted. The wind will only affect how soon you hear the sound.