Useful Equations

✔️ **Celsius-to-Fahrenheit:**  \( T_F = \frac{9}{5} T_C + 32 \)

✔️ **Celsius-to-Kelvin:**  \( T_K = T_C + 273.15 \)

✔️ **Thermal Expansion:**  \( \Delta L = \alpha L_0 \Delta T \);  \( \Delta V = \beta V_0 \Delta T \)

✔️ **Thermal Stress:**  \( \frac{F}{A} = -\alpha Y \Delta T \)  (For a linear expansion)

Problems:

1. (*) Convert the following temperatures:
   a) Room temperature is 70°F – What is this in Celsius?
      \[ T_C = \frac{5}{9} (T_F - 32) = \frac{5}{9} (70°) - 32 = 21°C \]
   b) Cast Iron melts at 1,375°C – What is this in Fahrenheit? (In Kelvin?)
      \[ T_F = \frac{9}{5} T_C + 32 = \frac{9}{5} (1,375°) + 32 = 2,507° F \ (1,648 K) \]
   c) At what temperature is the °F equal to the °C?
      \[ T = \frac{9}{5} T_C + 32 \rightarrow T \left( 1 - \frac{9}{5} \right) = 32 \rightarrow T = 32 - 0.8 = -40°C = -40° F \]

2. (**) A wire of length 1 meter (at room temperature) made of steel (coefficient of expansion 13 \( \times 10^{-6} \) \( K^{-1} \)) is cooled down to the freezing point of water. By how much does the length change? (The young's modulus of steel is 2 \( \times 10^{11} \) Pa)
   \[ \Delta L = \alpha L_0 \Delta T = 13 \times 10^{-6} K^{-1} \times 1 \text{ m} \times 20°C = 0.26 \text{ mm} \]

3. (**) On a cold morning (say 5°C) you fill your 15 gallon gas tank to the brim. (The volumetric coefficient of thermal expansion of gasoline is \( \beta = 9.5 \times 10^{-4} K^{-1} \)) If the hottest temperature that day was 24°C, how much gas would overflow out of your tank? (Assume that the change in volume of the tank is negligible).
   \[ \Delta V = \beta V_0 \Delta T = 9.5 \times 10^{-4} K^{-1} \times 15 \text{ gal} \times 24°C - 5°C = 0.27 \text{ gal} \]  (over a \( 1/4 \) gallon of gas!)
4. (***) Using the result from #3 – If your car is supposed to get 25 mpg in 24°C weather, what mileage will it get in 5°C weather? (Assume that the fuel injectors determine fuel usage by volume)

   • If a car gets 25 mpg, then with 15 gallons it will go $25 \text{ mpg} \times 15 \text{ gal} = 375 \text{ mi}$
   • If the same mass of gasoline is in your tank and it gets colder, then you will have $15 - 0.27 = 14.73 \text{ gal}$. So your car will only go a distance of $25 \text{ mpg} \times 14.73 \text{ gal} = 368 \text{ mi}$.
   • This makes it appear as though your mileage has reduced to $\frac{368 \text{ mi}}{15 \text{ gal}} = 24.5 \text{ mpg}$ (even if the mileage wouldn't change)

5. (***) The gas tank in part 3 is made the same steel as part 2 – recalculate your answer for part 3 without neglecting the change in volume of the gas tank. Was it acceptable to neglect it?

   • $\Delta V_{\text{steel}} = 3 \alpha V_0 \Delta T = 3 \left( 13 \times 10^{-6} \text{ K}^{-1} \right) \left| 15 \text{ gal} \right| \left| 19^\circ \text{C} \right| = 0.011 \text{ gal}$
   • (Compared to 0.27 gal increase in volume of gasoline means the overflow will only be $0.27 - 0.01 = 0.26$ gallons)
   • Neglecting the change in volume of the tank was an acceptable approximation.

6. (***) A wire of length 1 meter is fixed at both ends at room temperature. If this wire is made of the same steel as problem #2, what will be the tension in the wire if it is cooled to the freezing point of water? (Assume the cross-sectional area of the wire is 0.25 mm$^2$)

   • We know from the answer to question #2 that the change in length is $\Delta L = 0.26 \text{ mm}$.
   • We know that the Young's modulus $Y = \frac{F_A}{\Delta L / L_0}$, therefore the tension in the wire is

   $$ T = F_\perp = \frac{YA}{L_0} = \frac{(2 \times 10^{11} \text{ Pa})(0.26 \times 10^{-6} \text{ m}^2)(0.25 \times 10^{-6} \text{ m})}{1 \text{ m}} = 0.013 \text{ N} $$