Useful Equations

✔ Momentum: \( \vec{p} = m \vec{v} \); \( \vec{F}_{NET} = \frac{d \vec{p}_{NET}}{dt} \)

✔ Center of Mass: \( \vec{r}_{cm} = \frac{1}{M} \sum_i m_i \vec{r}_i \); \( \vec{r}_{cm} = \frac{1}{M} \int \vec{r} \, dm \)

Problems:

1. (***) On the grid below, mark (and calculate) the location of the center of mass.

   ![Grid Image]

   - Remember that the position is a vector.
   - \( \vec{r}_{cm} = \frac{1}{M} \sum_i m_i \vec{r}_i = \frac{1}{17} \left[ 5 \text{kg} \langle 0, 1 \rangle + 2 \text{kg} \langle 2, 1 \rangle + 10 \text{kg} \langle 1, 0 \rangle \right] \)
   - \( = \frac{1}{17} \langle 14 \text{kg} \cdot \text{m}, 7 \text{kg} \cdot \text{m} \rangle = \langle 0.824 \text{m}, 0.412 \text{m} \rangle \)

2. (***) On the object below, locate the center of mass.

   ![Object Image]

   - We can break this object into three pieces, each with its own center of mass. I placed my origin at the center of the middle rectangle.
   - A square 1 x 1 with cm position (-1.5, 1), a rectangle 1 x 4 with cm position (0,0), and a 1 x 2 rectangle with cm position (1, -1)
   - \( \vec{r}_{cm} = \frac{1}{M} \sum_i m_i \vec{r}_i = \frac{1}{7} \langle 1 \langle -1.5, 1 \rangle + 4 \langle 0, 0 \rangle + 2 \langle 1, -1 \rangle \rangle \)
   - \( = \frac{1}{7} \langle 0.5, -1 \rangle = \langle 0.0714, -0.143 \rangle \)
3. (***) Find the center of mass of a square plate of uniform density and side length \( s \) with a circular hole of radius \( s/4 \) in one corner. I decided to place my coordinates at the center of the square.
   - We can find the center of mass of an object by *subtracting* known objects as well.
   - The mass of the plate (due to its uniform density) is proportional to the area.
   - \( M_{sq} = \sigma A = \sigma s^2 \) And CM position \( \vec{r}_{sq} = \langle 0, 0 \rangle \)
   - \( M_{cir} = \sigma A = \sigma \pi r^2 = \sigma \pi \left( \frac{s}{4} \right)^2 \) and CM position \( \vec{r}_{cir} = \langle \frac{s}{4}, \frac{s}{4} \rangle \)
   - Thus the center of mass of the entire object is given by
     \[
     \vec{r}_{CM} = \frac{1}{\sigma s^2 + \frac{1}{16} \sigma \pi s^2} \left[ \sigma s^2 \langle 0, 0 \rangle - \frac{1}{16} \sigma \pi s^2 \langle \frac{s}{4}, \frac{s}{4} \rangle \right]
     \]
     \[
     = -\frac{1}{16} + \frac{\pi}{16} \frac{1}{s} \left\langle \frac{1}{4}, \frac{1}{4} \right\rangle = -0.1641 \ s \langle 0.25, 0.25 \rangle = s \langle -0.0410, -0.0410 \rangle
     \]

4. (*) Two particles are moving towards each other on a horizontal frictionless surface. One particle has a mass \( m \) and speed of \( 2v \), the other has a mass of \( 2m \) and speed \( v \). After the particles collide, what is the speed of the center of mass of the system?
   - We can see that the total momentum of the system is zero (the two particles have equal and opposite momenta).
   - We can also see that momentum must be conserved (because there are no net external forces). Therefore after *any* kind of collision the momentum must still be zero.
   - Because momentum is proportional to the velocity, the velocity of the system must always be zero.