Definition Review:

1) Classify each of the following differential equations by determining:
   i. Linear? Ordinary? What is the order? Independent Variable?
   
   a. \[ 25 \frac{d^2x}{dt^2} - 5 \frac{dx}{dt} - 10x = \sin(15t) \]
   
   b. \[ \frac{1}{y-1} y' = 2xy \]
   
   c. \[ \frac{\partial^2 u}{\partial x^2} = \frac{1}{v(x,t)} \frac{\partial^2 u}{\partial t^2} \]
   
   d. \[ \frac{e^x}{x^2y^2} (y')^2 + \left( \frac{x}{y} \right)^2 = \left( \frac{1}{y^2} \right) \int_1^x e^{t^2} dt \]

2) Consider the initial conditions for 1b. of the point \( y(0)=2 \).
   i. Does this IVP have a solution (at least in some interval around 0)? Is this solution unique?
   
   ii. Solve the IVP for an explicit solution \( \text{Hint: separable, partial fractions} \)
3) For what value of \( m \) is \( y = x^m \) A solution of the following DE?

i. \( x^2 y'' - 7xy' + 15y = 0 \)

4) Given: \( x = c_1 \cos t + c_2 \sin t \) is a two-parameter family of solutions of the second-order DE \( x'' + x = 0 \). Find a solution of the second order IVP given the initial conditions:

\[ x\left(\frac{\pi}{6}\right) = 0.5, \quad x'\left(\frac{\pi}{6}\right) = 0 \]

5) Newton’s Law of Cooling

5. A cup of coffee cools according to Newton’s law of cooling (3). Use data from the graph of the temperature \( T(t) \) in Figure 1.3.10 to estimate the constants \( T_m, T_0 \), and \( k \) in a model of the form of a first-order initial-value problem: \( \frac{dT}{dt} = k(T - T_m). \) \( T(0) = T_0. \)

![Figure 1.3.10](image.jpg)  
**FIGURE 1.3.10** Cooling curve in Problem 5
32. From \( y = x^m \) we obtain \( y' = mx^{m-1} \) and \( y'' = m(m-1)x^{m-2} \). Then \( x^2y'' - 7xy' + 15y = 0 \) implies

\[
x^2m(m-1)x^{m-2} - 7mx^{m-1} + 15x^m = [m(m-1) - 7m + 15]x^n
\]

\[
= [m^2 - 8m + 15]x^m = (m - 3)(m - 5)x^n = 0.
\]

Since \( x^n > 0 \) for \( x > 0 \), \( m = 3 \) and \( m = 5 \). Thus \( y = x^3 \) and \( y = x^5 \) are solutions.

9. From the initial conditions we obtain

\[
\frac{\sqrt{3}}{2} c_1 + \frac{1}{2} c_2 = \frac{1}{2}
\]

\[
-\frac{1}{2} c_1 + \frac{\sqrt{3}}{2} c_2 = 0.
\]

Solving, we find \( c_1 = \sqrt{3}/4 \) and \( c_2 = 1/4 \). The solution of the initial-value problem is

\[
x = (\sqrt{3}/4) \cos t + (1/4) \sin t.
\]

5. From the graph in the text we estimate \( T_0 = 180^\circ \) and \( T_n = 75^\circ \). We observe that when \( T = 85 \), \( dT/dt \approx -1 \). From the differential equation we then have

\[
k = \frac{dT/dt}{T - T_m} = \frac{-1}{85 - 75} = -0.1.
\]