SI Exam 1 Review

2. The following two lines are intersecting:

\[
\begin{align*}
x &= 1 - t \\
y &= 2 + t \\
z &= 3 - 2t
\end{align*}
\quad \text{and} \quad
\begin{align*}
x &= t \\
y &= 3 - t \\
z &= 3 + t
\end{align*}
\]

Find the line which goes through the intersection point and is perpendicular to both given lines.

5. Find the length a particle travels along the parametric curve \((\sin(2t), \frac{1}{2}t^2 - \ln t, \cos(2t))\) for \(1 \leq t \leq e\).

1. The surface \(\rho = 4 \cos \phi\) is a sphere. Identify the radius and center of the sphere and rewrite the equation in cylindrical coordinates.

2. Find the plane that contains the line

\[
\frac{x - 2}{3} = 2 - y = \frac{2z - 1}{4}
\]

and is perpendicular to the plane \(2x + 2y = z + \pi^3\).

4. You find yourself on an adventure with twelve dwarves traveling to (ahem) borrow Smaug’s treasure. As you are crossing the Misty Mountains your company finds themselves being chased by goblins on a mountain path. From your vast experience of being chased by Farmer Maggot back in Hobbiton you know that the best chance to evade capture is to hide by getting off the path at a point where the path sharply turns, i.e. at a point with high curvature and in particular a point on the road with \(\kappa > 10\) would give a point of escape. Glancing at a map of the path (which you fortunately tucked away in your bag) you see that the path you are on can be parameterized by

\[
\chi(t) = (5t^2 - 20t + 37, t^3 - 3t^2 + 41, 3t^2 - 11t + 137).
\]

[A little known fact is that Hobbiton prides itself on its multivariable calculus education program.]

Glóin, one of your dwarf companions, recommends getting off the path corresponding to the point when \(t = 2\); Nori, another dwarf, thinks that the road does not bend sharply enough at that point and that instead they should only try to outrun the goblins. Which plan of action should you take? Justify your answer.
4. (a) Find $a_T$ and $a_N$ when $r(t) = (e^t + 17, 2e^{-t} + \pi, 137 - 2t)$. Simplify any as much as possible.

4. Find the osculating plane to $r(t) = (e^{2t} - 3t, \sin(2t) - \cos(t), t^2 - 2t + 4)$ at $t = 0$. (Recall the osculating plane contains the point, the direction of motion, and the direction of acceleration.)

1. Find the vector projection of $u = (3, 2, 1)$ onto the vector $v = (1, 5, -3)$.

2. Find the equation of the plane perpendicular to the curve $x = t^5, y = 2t^2, z = 3t$ at $t = -1$.

1. Find parametric equations for the tangent line to the curve $x = 2t^2 + t, y = 2t^2, z = t^3 - t$, at $t = 2$.

2. Find parametric equations of the line of intersection of the planes

\[ 2x + 3y + 2z = 2 \]

and

\[ x + y + z = 3. \]