1) Volume:

a) Consider the region bounded by the graphs of \( y = \tan^{-1} x, y = 0, \) and \( x = 1.\) Find the volume of the solid formed by revolving this region about the y-axis.

\[
V = 2\pi \int_0^1 rh \, dx
\]

\[
= 2\pi \int_0^1 x \tan^{-1} x \, dx
\]

\[
= 2\pi \left[ \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \int_0^1 \frac{x^2}{1 + x^2} \, dx \right]
\]

\[
= 2\pi \left[ \frac{x^2 \tan^{-1} x}{2} - \frac{1}{2} \left( \frac{\pi}{4} - 0 \right) \right]
\]

\[
= 2\pi \left[ \frac{\pi}{4} - \frac{1}{2} \right]
\]

b) A solid has as its base a right triangle of lengths 3, 4, and 5 in the first quadrant with side length 4 extending across the x-axis and side length 3 extending along the y-axis. Cross-sections perpendicular to the side of length 4 (x-axis) are squares with one side of the square in the plane of the base. Find the volume of this solid.

\[
A = S^2
\]

\[
A(x) = \left( 3 - \frac{3}{4} x \right)^2
\]

\[
\Rightarrow V = \int_0^4 \left( 3 - \frac{3}{4} x \right)^2 \, dx
\]

\[
= \int_0^4 \left( 9 - \frac{9}{2} x + \frac{9}{16} x^2 \right) \, dx
\]

\[
= 9x - \frac{9}{2} \frac{x^2}{2} + \frac{9}{16} \frac{x^3}{3} \bigg|_0^4
\]

\[
= 9(3) - \frac{9}{4}(16) + \frac{3}{16}(64) = 12
\]
2) Arc Length & Surface Area:
   a) For the circle \( x^2 + y^2 = 25 \), rotated about the x-axis, set up an integral to find the surface area of the solid from \( x = -2 \) to \( 3 \).

\[
SA = 2\pi \int_{-2}^{3} y \sqrt{1 + \left( \frac{dy}{dx} \right)^2} \, dx
\]

\[
y = \sqrt{25 - x^2}
\]

\[
y' = \frac{-x}{\sqrt{25 - x^2}}
\]

\[
\int_{-2}^{3} 2\pi \sqrt{25 - x^2} \sqrt{1 + \left( \frac{-x}{\sqrt{25 - x^2}} \right)^2} \, dx
\]

3) Centroids:
   a) Let \( R \) be the region between \( y = x^2 \) & \( y = \sqrt{x} \) from \( x = 0 \) to \( 1 \), which has a density function \( \delta(x) = x^2 \). Find the center of mass for \( R \).

\[
M_y = \int_0^1 8(x) \left[ f(x) - q(x) \right] \, dx
\]

\[
M_x = \int_0^1 8(x) \left[ f(x) - q(x) \right] \, dx
\]

\[
m = \int_0^1 8(x) \left[ f(x) - q(x) \right] \, dx
\]

\[
M_y = \int_0^1 x^2(x) \left[ \sqrt{x} - x^2 \right] \, dx = \int_0^1 x^{5/2} - x^5 \, dx = \frac{2}{3} x^{7/2} - \frac{1}{6} x^6 \bigg|_0^1 = 7 \frac{1}{18}
\]

\[
M_x = \int_0^1 x^2 \left[ \left( \sqrt{x} \right)^2 - (x)^2 \right] \, dx
\]

\[
= \int_0^1 x^3 - x^6 \, dx = \frac{1}{2} \left[ \frac{1}{4} x^4 - \frac{1}{7} x^7 \right] \bigg|_0^1 = \frac{1}{2} \left[ \frac{1}{4} - \frac{1}{7} \right] = \frac{3}{56}
\]

\[
m = \int_0^1 x^2(\sqrt{x} - x^2) \, dx = \int_0^1 x^{5/2} - x^4 \, dx = \frac{2}{7} x^{7/2} - \frac{1}{5} x^5 \bigg|_0^1
\]

\[
(\bar{x}, \bar{y}) = \left( \frac{35}{34}, \frac{5}{8} \right)
\]
4) Fluid Work:

a) An exotic aquarium tank has height 16 ft. At a height \( x \) from the bottom of the aquarium the area of the cross-section perpendicular to the height is \( 4x^2 + 2 \) square feet. The aquarium is half full (filled to a depth of 8 ft) of water with a weight density of 62.4 pounds per cubic foot. Find the work done in pumping the water in the tank to the top.

\[
\begin{align*}
W & = \Delta U(8) \text{(distance)} \\
& = \Delta A(4x) \times 8 \text{(distance)} \\
& = (4x^2 + 2)(4x)(62.4)(16-3) \\
& = \int_0^8 (4x^2 + 2)(16-x)(62.4) \, dx = 62.4 \int_0^8 64x^2 + 32x + 32 \, dx \\
& = 62.4 \left[ \frac{-x^4 + 64}{3} x^3 - x^2 + 32x \right]_8 \\
& \approx 437,965 \text{ ft-lb}
\end{align*}
\]

b) For a 50 ft long tank with semi-circular ends of radius of 10 ft, which is filled with 7 ft of water, find the work done to pump all of the water out of the tank.

\[
\begin{align*}
W & = \Delta U(8) \text{(distance)} \\
& = \int_{-10}^3 8(50)(2)\sqrt{100-y^2} (-y) \, dy \\
& = 62.4(100) \int_{-10}^3 -y \sqrt{100-y^2} \, dy \\
& = 100(62.4) \int_{-10}^3 \frac{u^{1/2}}{2} \, du \\
& = \frac{100(62.4)}{2} \left[ \frac{u^{3/2}}{3} \right]_{-10}^3 \\
& \approx 1865, 620 \text{ ft-lb}
\end{align*}
\]
1) Integration:

a) \[
\int \sqrt{x} \ln(2x^2) \, dx = \int u \, dv = \frac{2}{x} \, dx = \sqrt{x} \, dx
\]

\[
= \frac{2}{3} x^{3/2} \ln(2x^2) - \frac{2}{3} \int \frac{2}{3} x^{3/2} \left( \frac{2}{x} \right) \, dx
\]

\[
= \frac{2}{3} x^{3/2} \ln(2x^2) - \frac{8}{9} x^{3/2} + C.
\]

b) Consider the integral \[
\int \frac{Ax^2 + Bx + 1}{(4x^3 - 6x^2 + 2x - 3)^3} \, dx
\]
pick values for A and B such that this is an easy integral to evaluate and then perform the integration.

Let \( A = 6 \) \( B = -6 \) (Making easy \( u \)-sub.)

So

\[
\int \frac{6x^2 - 6x + 1}{(4x^3 - 6x^2 + 2x - 3)^3} \, dx
\]

\[
u = 4x^3 - 6x^2 + 2x - 3
\]

\[
dv = 12x^2 - 12x + 2 \, dx
\]

\[
= \frac{1}{2} \int u^{-7/3} \, du = \frac{1}{2} \left( \frac{3}{4} \right) u^{-4/3} + C
\]

\[
= \frac{3}{8} \left( 4x^3 - 6x^2 + 2x - 3 \right)^{-4/3} + C
\]
2) Integration:

a) \[
\int_0^{\pi/12} \tan^3(3\theta) \sec^4(3\theta) d\theta
\]

\[
= \frac{1}{3} \left[ \frac{1}{4} u^4 + \frac{1}{6} u^6 \right]_{\theta=0}^{\beta=\pi/12}
\]

\[
= \frac{1}{12} \tan^4(\beta) + \frac{1}{18} \tan^6(\beta) \bigg|_0^{\pi/12}
\]

\[
= \frac{1}{12} \tan^4\left(\frac{\pi}{24}\right) + \frac{1}{18} \tan^6\left(\frac{\pi}{24}\right) = \frac{5}{36},
\]

b) \[
\int \frac{x}{(16 - x^4)^{\frac{3}{2}}} dx
\]

\[
= \int \frac{4 \cos \theta}{\sqrt{16 - 16 \sin^2 \theta}} d\theta
\]

\[
= \int \frac{2 \cos \theta}{\sqrt{4 \cos^2 \theta}} d\theta = \frac{1}{2} \int \frac{\cos \theta}{\cos \theta} d\theta = \frac{1}{2} \int d\theta = \frac{1}{2} \theta + c
\]

\[
= \frac{1}{2} \sin^{-1}\left(\frac{x^2}{2}\right) + c
\]
\[ c) \int \frac{5x - 4}{4x^2 + 9x + 2} \, dx = \frac{5x - 4}{(4x+1)(x+2)} = \frac{A}{x+2} + \frac{B}{4x+1} \]

So \( A(4x+1) + B(x+2) = 5x-4 \)

\[ \begin{align*}
 x &= -2, & A &= 2 \quad & \Rightarrow & & \frac{2}{x+2} - \frac{3}{4x+1} \\
 x &= -\frac{1}{4}, & B &= -\frac{21}{4} \quad & \Rightarrow & & \frac{7}{4} \quad & \Rightarrow & & B = -3
\end{align*} \]

\[ \int \frac{2}{x+2} - \frac{3}{4(x+1)} \, dx = 2 \ln |x+2| - \frac{3}{4} \ln |4x+1| + C \]