\[ \sin 7x - \sin x = \]

\[ f(x) = \frac{5}{3 - 2x} \]

Use the definition of the derivative as the limit of a difference quotient to compute \( f'(x) \).
No credit for use of the short rules.

\[ \lim_{h \to 0} \frac{\frac{5}{3 - 2(x+h)} - \frac{5}{3 - 2x}}{h} = \frac{5(3 - 2x) - 5(3 - 2(x+h))}{(3 - 2(x+h))(3 - 2x)} \quad \xrightarrow{h \to 0} \quad \frac{5}{(3 - 2x)^2} = f''(x) \]

(5 points) Find

\[ \lim_{x \to 0} \sqrt{10x^4 + 5x^3 + 1} = \sqrt{\frac{10}{2}} = \sqrt{5} \]

Give the exact answer.

(5 points) Find

\[ \lim_{x \to 0} \frac{\sin 3x \cos 4x}{\sin 5x} \]

Give the exact answer.

\[ \lim_{x \to 0} \cos 4x \quad \lim_{x \to 0} \frac{\sin 3x}{\sin 5x} \quad \lim_{x \to 0} \frac{5}{3} \frac{x}{\sin 5x} \cdot \frac{\sin 3x}{3} = \frac{3}{5} \]
Question 6. The graph $y = F(t)$ of a function $F$ is shown below. Evaluate $\lim_{t \to 1} \left( (2t^2 - 3)F(t) \right)$ or give reasons why the limit does not exist.

\[
\begin{align*}
\lim_{t \to 1} (2t^2 - 3)F(t) &= 2 \\
\lim_{t \to 4} 2t^2 - 3 &= 29 \\
\lim_{t \to 4} f(t) \cdot \lim_{t \to 4} (2t^2 - 3) &= 58
\end{align*}
\]

Question 6. Write a formula for a function $f$ so that the graph of $y = f(x)$ has horizontal asymptote $y = 2$, vertical asymptotes at $x = -1$ and $x = 3$, and no other asymptotes. You must show that your function has the required asymptotes. (10 Points.)

\[
f(x) = \frac{2x^2}{(x+1)(x-3)} \to \frac{2x^2}{x^2 - 2x - 3}
\]
Find the equation of the line tangent to the graph of \( y = \sqrt{x^2 + 4x - 5} \) at \( x = 3 \). Simplify your answer.

\[
\lim_{h \to 0} \frac{\sqrt{(x+h)^2 + 4(x+h) - 5} - \sqrt{x^2 + 4x - 5}}{h}
\]

\[
\lim_{h \to 0} \frac{(x+h)^2 + 4(x+h) - 5 - (x^2 + 4x - 5)}{h}
\]

\[
\lim_{h \to 0} \frac{2x + h^2 + 4h}{h}
\]

\[
\lim_{h \to 0} \frac{2x + h^2 + 4h}{\sqrt{(x+h)^2 + 4(x+h) - 5} + \sqrt{x^2 + 4x - 5}} = \frac{2x + 4}{2\sqrt{x^2 + 4x - 5}}
\]

Plug in \( x = 3 \).
Evaluate
\[
\lim_{r \to 3} \frac{r^2 - 9}{\sqrt{r} - \sqrt{3}} = \frac{0}{0}, \quad \frac{r^2 - 9}{\sqrt{r} - \sqrt{3}} \cdot \frac{\sqrt{r} + \sqrt{3}}{\sqrt{r} + \sqrt{3}} = \frac{(r-3)(r+3)(\sqrt{r} + \sqrt{3})}{\sqrt{3} - 3}
\]

\[
\lim_{r \to 3} \frac{(r+3)(\sqrt{r} + \sqrt{3})}{1} = 6(2\sqrt{3}) = 12\sqrt{3}
\]

**Question 5.** Find the equation of the line tangent to the graph of \( y = \frac{2\ln x}{x^2 + 1} \) at the point on the graph with \( x = 1 \).