4.1 Applications of the first derivative

**Determining intervals where a function increases or decreases**

1. Find all values of \( x \) for which \( f' \) is 0 or DNE and identify the open intervals determined by these numbers.
2. Select a test number \( 'c' \) in each interval found in step 1 and determine the sign of \( f'c \) in that interval
   a. If \( f' \) greater than 0, \( f \) is increasing on that interval
   b. If \( f' \) less than 0, \( f \) is decreasing on that interval

**Relative Maximum**

A function \( f \) has a relative max at \( 'c' \) if there exists an open interval \((a,b)\) containing \( 'c' \) such that \( f(x) \leq f(c) \) for all \( x \) in \((a,b)\)

**Relative minimum**

A function has a relative min at\( 'c' \) if there exists an open interval \((a,b)\) containing \( c \) such that \( f(x) \geq f(c) \) for all \( x \) in \((a,b)\)

**Critical Number**

A critical number of a function \( f \) is any number \( x \) in the domain of \( f \) such that \( f'(x)=0 \) or DNE

4.2 Applications of the second derivative

**Determining Concavity**

1. Determine the values of \( x \) for which \( f'' \) is zero or DNE and identify the open intervals determined by these numbers
2. Determine the sign of \( f'' \) in each interval found in step 1. Compute \( f'' \) for any conveniently chosen test number.
   a. If \( f'' > 0 \) then the graph is concave upward
   b. If \( f'' < 0 \) then the graph is concave downward

**Finding inflection points**

1. Compute \( f''(x) \)
2. Determine the numbers in the domain of \( f \) for which \( f''=0 \) or DNE
3. Determine the sign of \( f'' \) to the left and right of each number found in step 2. If there is a change in sign of \( f'' \) moving across \( c \), then \((c,f(c))\) is an inflection point

**Second Derivative Test**

1. Compute \( f' \) and \( f'' \)
2. Find all the critical numbers of \( f \) at which \( f'=0 \)
3. Compute \( f''(c) \) for each critical number \( c \)
   a. If \( f''(c)<0 \) then \( f \) has a relative max at \( c \)
   b. If \( f''(c)>0 \) then \( f \) has a relative min at \( c \)
   c. If \( f''(c)=0 \) or DNE the test fails
4.3 Curve sketching

**Finding vertical asymptote**

Where ever denominator of function =0 and numerator ! =0

**Finding horizontal asymptote**

The line $y=b$ is a horizontal asymptote if $\lim_{x \to \pm \infty} f(x) = b$

**Guide to curve sketching**

1. Determine the domain of $f$
2. Find the $x$ and $y$ intercepts (may neglect $x$ intercept)
3. Determine the behavior of $f$ for large absolute values of $x$
4. Find all horizontal/vertical asymptotes
5. Determine the intervals where $f$ is increasing or decreasing
6. Find the relative min/max of $f$
7. Determine the concavity
8. Find inflection points
9. Plot a few additional points to help identify shape