Useful Equations

\[ \tau = IBA \sin \phi \]  
Magnitude of torque on current

\[ \mu = IA \]  
Magnetic dipole

\[ \overrightarrow{\tau} = \overrightarrow{\mu} \times \overrightarrow{B} \]  
Torque as a cross product

\[ U = -\overrightarrow{\mu} \cdot \overrightarrow{B} \]  
Potential energy due to magnetic field

Related Problems

1) The plane of a rectangular loop of wire with a width of 5.0 cm and a height of 8.0 cm is parallel to a magnetic field of magnitude 0.21 T. The loop carries a current of 6.6 A. (Book 27.42)
(a) what torque acts on the loop?
\[ \tau = IBA \sin \phi = IBwh \sin(90) = 0.0055 \, Nm \]

(b) What is the magnetic moment of the loop?
\[ \mu = IA = lwh = 0.026 \, Am^2 \]

(c) What is the maximum torque that can be obtained with the same total length of wire carrying the same current in this magnetic field?
\[ \tau = \frac{IB(w+h)^2}{\pi} = 7.5(10^{-3}) \, Nm \]

2) A uniform rectangular coil of total mass 270 g and dimensions 0.5m x 1.0m is oriented perpendicular to a uniform 3.60-T magnetic field. A current of 2.60 A is suddenly started in the coil. (Book 27.45)
(a) About which axis (A_1 or A_2) will the coil begin to rotate?
Since \( \tau = IBA \sin \phi \) then only the top and bottom sides will experience torque, thus will rotate about A_2

(b) Find the initial angular acceleration of the coil just after the current is started.
Using Newton’s Second law,
\[ \sum \tau = I\alpha \]

Now, each 1.00-m side has a mass of 0.090 kg. And each 0.500-m side has a mass of 0.045 kg.
To find the moment of inertia of the coil, consider each side as a uniform bar. The 2 0.5-m sides rotate about their centers, and the 2 1.0-m sides rotate about a parallel axis.

Thus,
\[ I = 2(0.090)(0.250)^2 + 2 \frac{1}{12} (0.045)(0.500)^2 = 0.013125 \text{ kgm}^2 \]

\[ |\tau| = IAB \sin(90) = 4.68 \text{ N} \]

\[ \tau = I \alpha \rightarrow \alpha = \frac{\tau}{I} = 356 \text{ rad/s}^2 \]

3) A coil with a magnetic moment of 1.46 Am^2 is oriented initially with its magnetic moment antiparallel to a uniform magnetic field of magnitude 0.825 T. What is the change in potential energy of the coil when it is rotated 180 degrees so that its magnetic moment is parallel to the field? (Book 27.47)

\[ U = \mu B \cos \phi \]

\[ U_1 = \mu B \cos(0) = \mu B \]

\[ U_1 = \mu B \cos(180) = -\mu B \]

\[ \Delta U = U_2 - U_1 = -2\mu B = -2.41 \text{ J} \]