MATH 267 SI Exam Review

Topics
1. Find and classify critical points of an autonomous differential equations
2. Separable differential equations
3. Linear differential equations
4. Exact differential equations
5. Solve a differential equation using an appropriate substitution
6. Use a first-order differential equation for modeling (population, heating/cooling, mixtures)

Given

- Given a linear first-order differential equation in standard form
  \[ \frac{dy}{dx} + P(x)y = f(x), \]
  then
  \[ \mu(x)y = \int \mu(x)f(x) \, dx, \]
  where \( \mu(x) \) is the integrating factor.

- For an exact first-order differential equation
  \[ M(x, y) \, dx + N(x, y) \, dy = 0, \]
  an implicit solution is given by
  \[ C = f(x, y), \]
  where
  \[ \frac{\partial f}{\partial x} = M(x, y) \quad \text{and} \quad \frac{\partial f}{\partial y} = N(x, y). \]

- Given a homogeneous first-order differential equation
  \[ M(x, y) \, dx + N(x, y) \, dy = 0, \]
  make the substitution \( y = ux. \)

- For a first-order differential equation of the form
  \[ \frac{dy}{dx} = f(Ax + By + C) \]
  make the substitution \( u = Ax + By + C. \)
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**PROBLEM 1**

2. Find the critical points, phase portrait and classify each critical point as asymptotically stable, unstable or semi stable for the differential equation \( \frac{dy}{dx} = y^3 - y^2 - 6y \). Also draw typical solution curves.

**PROBLEM 2**

2. Find the critical points, phase portrait and classify each critical point as asymptotically stable, unstable or semi stable for the differential equation \( \frac{dy}{dx} = y^2 \ln y^2 - y^2 \). Also draw typical solution curves.

**PROBLEM 3**

**Problem 1** (25 points)

a) Find in explicit form the general solution of the differential equation

\[
\frac{dy}{dx} = -(1 + x^2)y^2
\]

b) Solve the boundary value problem

\[
y' - xy = -x, \quad y(0) = 0
\]

**PROBLEM 4**

2. (15pts) Solve the following IVP

\[
y' + 2xy = e^{-x^2}, \quad y(0) = 1.
\]

**PROBLEM 5**

4. Use an appropriate method to solve \( y' + \tan x \cdot y = \cos x, \quad y(0) = 1 \).

**PROBLEM 6**

3. (15pts) Find the general solution of the following exact equation in implicit form

\[
2xe^y dx + (x^2 e^y + 1)dy = 0.
\]
PROBLEM 7

Problem 1 (25 points)
Find the general solution in explicit form of the following differential equation
\[ \left( 2 + \frac{1}{x} \right) dx - \frac{1}{y^2 x} dy = 0. \]

PROBLEM 8

5. (15pts) Use substitution to transform the following homogeneous equation into a separable equation of the form \( \frac{du}{dx} = h(u)g(x) \). DO NOT solve the equation but only write down the resulting equation
\[ \left( x^2 + \frac{y^3}{x} \right) dx + y^2 dy = 0. \]

PROBLEM 9

6. (Bonus - 10 points): Solve the following Differential equation using appropriate substitution.
\[ \frac{dy}{dx} = (x + y + 1)^2 - (x + y - 1)^2 \]

PROBLEM 10

6. Solve Bernoulli differential equation \( x^2 \frac{dy}{dx} - 2xy = 3y^4 \), \( y(1) = \frac{1}{2} \).

PROBLEM 11

4. A roast turkey is taken from an oven when its temperature has reached 185°F and is placed on a table in a room where the temperature is 75°F. If the temperature of the turkey is 150°F after half an hour, When will the turkey have cooled to 100°F?

PROBLEM 12

5. The half-life of Cesium-137 is 30 years. Suppose we have a 100 mg sample. (a) Find the mass that remains after t years. (b) How much of the sample remains after 100 years? (c) After how long will only 1 mg remain?