2. A particle’s position (in meters) at time $t$ (seconds) is given by the function

$$s(t) = \frac{4}{t-2} + a \sin t + \frac{1}{3} be^{3t}, \quad \text{for} \quad -1 < t < 1,$$

where $a$ and $b$ are constants. Find $a$ and $b$ so that at time $t = 0$ the velocity is $0 \text{ m sec}^{-1}$ and acceleration is $0 \text{ m sec}^{-2}$.

$$s'(t) = 4(t-2)^{-2} + as \cos t + \frac{1}{3} be^{3t}$$

$$s''(t) = 4(t-2)^{-2} - a \sin t + 3be^{3t}$$

where

$$s'(0) = -1 + a + b = 0$$

$$s''(0) = \frac{8}{(0-2)^2} - a \sin(0) + 3be^{3(0)} = \frac{8}{4} - 0 + 3b$$

Thus

$$b = \frac{2}{3}$$

and

$$a = \frac{2}{3}$$
Using the limit definition of the derivative, find the derivative of \( f(x) = -\frac{2}{2x-1} \)
(No credit will be given for the derivative found by any other method.)

\[
\lim_{h \to 0} \frac{-\frac{2}{2(x+h)-1} + \frac{2}{2x-1}}{h} = \lim_{h \to 0} \frac{\frac{-2(2x-1) + 2(2(x+h)-1)}{[2(x+h)-1][2x-1]}}{h} = \lim_{h \to 0} \frac{-4x + 2 + 2(2x+2h-1)}{[2(x+h)-1][2x-1]}
\]

\[
= \lim_{h \to 0} \frac{-4x + 2 + 4x + 4h - 2}{h[2(x+h)-1][2x-1]} = \lim_{h \to 0} \frac{4h}{h[2(x+h)-1][2x-1]} = \lim_{h \to 0} \frac{4}{2(x+h)-1} = \frac{4}{2x-1}
\]

\[
f'(x) = \frac{-2(2x-1)^2 + 4}{(2x-1)^2} = \frac{4}{(2x-1)^2}
\]
5. The function $f(x)$ is a piecewise function given as a formula below on the left; the function $g(x)$ is a different piecewise function given as a graph below on the right.

\[ f(x) = \begin{cases} 
  e^{\sin(\frac{\pi}{2}x)} & \text{if } x \leq 0 \\
  x \cos(\frac{\pi}{4}x) & \text{if } x > 0 
\end{cases} \]

(a) If $R(x) = \sqrt{g(x)}$, find $R'(0)$ or explain why it does not exist.

\[ R(x) = (g(x))^{1/2} \]
\[ \frac{dR}{du} = \frac{1}{2} u^{-1/2} \]
\[ R'(x) = \frac{1}{2} \frac{dR}{du} \cdot \frac{du}{dx} = \frac{1}{2} \frac{dR}{du} \cdot \frac{d}{dx} \]
\[ R'(0) = \frac{2}{2 \sqrt{2}} = \frac{1}{\sqrt{2}} \]

(b) If $S(x) = f(x)g(x)$, find $S'(-3)$ or explain why it does not exist.

\[ S(x) = e^{\sin(\pi x)} \]
\[ S'(-3) = \]
\[ S'(-3) = \frac{e^{-3}}{2} \]

(c) If $T(x) = f(g(x))$, find $T'(3)$ or explain why it does not exist.

\[ T(u) = f(u) \]
\[ T'(x) = f'(u) \cdot g'(x) \]
\[ f'(u) = e^u \]
\[ g'(x) = \sin x \]
\[ f'(2) = e^{\frac{\pi}{2}} = \frac{\pi}{2} \]
\[ T'(3) = f'(g(3)) \cdot g'(3) \]
\[ f'(2) = e^{\frac{\pi}{2}} = \frac{\pi}{2} \]
\[ T'(3) = \]
\[ T'(3) = \frac{\pi}{2} \]

\[ f'(2) = -1 \]
\[ f'(2) = f'(-2) = \frac{\pi}{2} \]
\[ f'(2) = -1 \]
\[ f'(2) = -1 = -\frac{\pi}{2} \cdot -2 = \pi \]
The line $y = 3x + 1$ is tangent to the curve $y = f(x)$ at $x = 2$.
The line $x + y = 5$ is tangent to the curve $y = g(x)$ at $x = 3$
Find the tangent line to the curve $y = f(g(x))$ at $x = 3$

<table>
<thead>
<tr>
<th>Tangent Line</th>
<th>Curve</th>
<th>Point</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = 3x + 1$</td>
<td>$y = f(x)$</td>
<td>$(2, f(2))$</td>
<td>$f'(2) = 3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$(2, 7)$</td>
<td></td>
</tr>
<tr>
<td>$x + y = 5$</td>
<td>$y = g(x)$</td>
<td>$(3, g(3))$</td>
<td>$g'(3) = -1$</td>
</tr>
<tr>
<td>$y = 5 - x$</td>
<td></td>
<td>$(3, 2)$</td>
<td></td>
</tr>
</tbody>
</table>

$y - 7 = -3(x - 3)$

$y = f(g(x))$  
$(3, f(g(3)))$  
$f'(g(3)) \cdot g'(3) =$  
$(3, f(2))$  
$f'(2)g'(3) = 3 \cdot -1 = -3$  
$(3, 7)$
6. You have recently taken up a job on Moon 13 where you maintain and operate a machine that takes as input material blargs (b) and will produce as output material furbls (f). This particular machine will take b blargs and produce

$$f(b) = e^{2b} - 36e^b + 8b + \pi^3$$ (furbls).

You are trying to determine the current amount of input, b, being fed into the machine, so you refer to a monitor which usually displays information about the value of b (blargs), the value of f(b) (furbls), as well as rates coming from the derivative, the second derivative, the third derivative, and so on, each readout including appropriate units. You discover that the monitor is mostly broken, and the only thing which you can read currently states

```
furbls  = 88
blarg
```

Use this information to determine the current amount of blargs (b) being fed into the machine, and write your answer in the provided box. (Hint: $$e^{2b} = (e^b)^2$$.)

$$f'(b) = 2e^{2b} - 36e^b + 8$$

$$f''(b) = 4e^{2b} - 36e^b$$

Find a point where $$f''(b) = 88$$

$$88 = 4e^{2b} - 36e^b$$

$$22 = e^{2b} - 9e^b$$

$$e^{2b} - 9e^b - 22 = 0$$

$$(e^b - 11)(e^b + 2) = 0$$

$$e^b + 2 = 0, e^b = -2$$ not possible

$$e^b = 11$$

$$b = \ln 11$$
6. This problem concerns the following piece-wise function.

\[ f(x) = \begin{cases} 
  x + 3 & x < -3 \\
  -10 & x = -3 \\
  \frac{(x + 2)^2 + x - 4}{x - 2x^2} & -3 < x < 0 \\
  2 & x = 0 \\
  \frac{\sin(5x)}{\tan(x) \cos(x)} & 0 < x \leq \pi/4 
\end{cases} \]

Without using L'Hôpital’s rule, answer the following questions.

(a) Determine whether \( f(x) \) is continuous, has a jump discontinuity, or a removable discontinuity at \( x = -3 \). (Justify your answer using limits.)

\[
\lim_{x \to -3} (x + 3) = -3 + 3 = 0
\]

\[
\lim_{x \to -3^+} \frac{(x+2)^2 + x - 4}{x - 2x^2} = \frac{(-3+2)^2 + (-3) - 4}{-3 - 2(-3)^2} = 1 - \frac{7}{-3 - 2(9)} = \frac{-6}{-3 - 18} = \frac{-6}{-21} = \frac{2}{7}
\]

Limits don't match \( \Rightarrow \) jump discontinuity

(b) Determine whether \( f(x) \) is continuous, has a jump discontinuity, or a removable discontinuity at \( x = 0 \). (Justify your answer using limits.)

\[
\lim_{x \to 0^-} \frac{(x+2)^2 + x - 4}{x - 2x^2} = \lim_{x \to 0^-} \frac{x^2 + 4x + 4 + x - 4}{x(1 - 2x)} = \lim_{x \to 0^-} \frac{x(x + 5)}{x(1 - 2x)} = 5
\]

\[
\lim_{x \to 0^+} \frac{\sin(5x)}{\tan(x) \cos(x)} = \lim_{x \to 0^+} \frac{\sin(5x)}{\tan(x) \cos(x)} = \lim_{x \to 0^+} \frac{\sin(5x) \cdot \sin(x)}{\cos(x) \cdot 5x} = \lim_{x \to 0^+} \frac{5x}{x} = 5
\]

\[ f(0) = 0 \neq 5 = \lim_{x \to 0} f(x) \] Removeable discontinuity

(c) If the answer to (a) and/or (b) was a removable discontinuity, how should the function be defined at the point(s) to make it continuous?

\[ f(0) = 5 \]