1. \( f(x) = 7^x - 7^{-x} \)  
   \( g(x) = 4 \cdot 7^{2x} - 7^{2x} = 4 \cdot 7^{4x} \)  
   \( f(x)^2 + \sqrt{g(x)} = 1 \cdot 7^{2x} + 7^{-2x} - 2 \cdot 7^x + 2 \cdot 7^{2x} \)  
   \( = 3 \cdot 7^{2x} + 7^{-2x} - 2 \)

2. Find a log function \( y = c + b \log_a x \) that passes through \((1, 4)\) and \((2, 6)\)  
   \[ 4 = c + b \log_a 1 \implies 4 = c \]  
   \[ 6 = 4 + b \log_a 2 \implies 2 = b \log_a 2 \]  
   \[ a^2 = 2^b \implies a = 2, b = 2 \]  
   \[ y = 4 + 2 \log_2 x \]

3. Find the exponential through \((0, 2)\) and \((3, 54)\) \(y = ca^x\)  
   \[ 2 = ca^0 \implies 2 = c \]  
   \[ 54 = 2a^3 \implies 27 = a^3 \implies a = 3 \]

4. Solve \( 8^{\frac{2}{3}x+4} = 2 \)  
   \( x^2 - 3 = 2x + 12 \implies x^2 - 2x - 15 = 0 \)  
   \( (x-5)(x+3) = 0 \)  
   \( x = 5, x = -3 \)
\[(2x+3)(x+1)\]
\[2x^2 + 5x + 3\]

What values can \(x\) take on?

\[2e^{2x} + 5e^{x} + 3 = 0\]  \(e^{x} = y\)

\[2y^2 + 5y + 3 = 0\]  \(\alpha, \beta = 2.3, -6\)

\[2y^2 + 2y + 3 = 0\]  \(\beta = 3^{\frac{1}{2}}\)

\[2y(y+1) + 3(y+1) = 0\]

\[(y+1)(2y+3) = 0\]

\[e^{x+1} = 0 \Rightarrow e^{x} = -1\]  No Real Solutions

\[2e^{x+3} = 0 \Rightarrow e^{x} = -\frac{3}{2}\]

Find \(k\) if \(7^{4x} = 3^{kx}\)

\[\ln 7^{4x} = \ln 3^{kx} \Rightarrow 4x \ln 7 = kx \ln 3\]

\[4 \ln 7 = k \ln 3 \Rightarrow k = \frac{4 \ln 7}{\ln 3}\]

\[7 \left( \log_2 (x) + \log_2 (4x) \right) = 4\]

\[\log_2 (4x^2) = 4 \Rightarrow 2^4 = 4x^2 = 16 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2\]

8. \((\log (3x)) - 2 = \log (3x)\)  \((\log (3x)) - 2 - \log (3x) = 0\)

\[\log (3x) \left( \log (3x) - 2 \right) = 0\]

\[\log (3x) = 0 \Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3}\]

\[\log (3x) - 1 = 0 \Rightarrow \log (3x) = 1 \Rightarrow 10^1 = 10 \Rightarrow 3x = 10 \Rightarrow x = \frac{10}{3}\]

9. What happens to \(f(x) = \frac{1-e^{-x}}{1-x}\) as \(x\) approaches 0 and \(\infty, -\infty\)?

\[f(x) = \frac{20}{1-e^{-\infty}} = \frac{20}{1} = 20\]

\[f(x) = \frac{20}{1-e^{-\infty}} = \frac{20}{1} = 20\]

\[f(x) = \frac{20}{1-e^{-\infty}} = \frac{20}{1-\infty} = 0\]
10. Solve for $x$

$$\ln(5x-2) - \ln(7x-8) = 0$$

$$\ln \left(\frac{5x-2}{7x-8}\right) = 0 \Rightarrow \frac{5x-2}{7x-8} = 1$$

$$5x-2 = 7x-8 \Rightarrow 2x = 6 \Rightarrow x = 3$$

11. Find $f^{-1}(x)$

$$f(x) = \frac{2}{3} \log \left(\frac{x-2}{x+2}\right) = \log \left(\frac{x-2}{x+2}\right)^{2/3}$$

$$x = \log \left(\frac{y-2}{y+2}\right)^{2/3} \Rightarrow 10^x = \left(\frac{y-2}{y+2}\right)^{2/3} \Rightarrow 10^x = \frac{y-2}{y+2}$$

$$(y+2)10^{3x/2} = y-2 \Rightarrow y10^{3x/2} + 2 \cdot 10^{3x/2} = y-2$$

$$y \cdot 10^{3x/2} - y = -2 - 2 \cdot 10^{3x/2} \Rightarrow y(10^{3x/2} - 1) = -2 - 2 \cdot 10^{3x/2}$$

$$y = \frac{-2 - 2 \cdot 10^{3x/2}}{10^{3x/2} - 1}$$

13. Solve for $x$

$$\log_3 \left(\sqrt{x+3}\right) - \log_3 \left(\sqrt{3x-1}\right) = \frac{1}{2}$$

$$\log_3 \left(\sqrt{x+3}\right) = \frac{1}{2} + \log_3 \left(\sqrt{3x-1}\right)$$

$$\log_3 \left(\frac{x+3}{3x-1}\right) = 1$$

$$\frac{x+3}{3x-1} = 3 \Rightarrow x+3 = 9x-3$$

$$-8x = 6 \Rightarrow x = \frac{3}{4}$$

12. Find $f^{-1}(x)$

$$f(x) = 6x + 2$$

$$x = 6^y + 2$$

$$x-2 = 6^y \Rightarrow \ln(x-2) = \ln 6^y = y \ln 6$$

$$y = \frac{\ln(x-2)}{\ln 6}$$
14. Find the area of a sector $\theta = 40^\circ$ and $s = 6$ km.

Take $\pi = 3$.  

\[
A = \frac{r^2\theta}{2} \quad \text{and} \quad \theta = \frac{s}{r}
\]

\[
\theta = 40^\circ \cdot \frac{\pi}{180^\circ} = \frac{2\pi}{9} \quad \text{radians} \approx \frac{2.3}{9} = \frac{2}{3}
\]

\[
r = \frac{s}{\theta} = \frac{6}{\frac{2}{3}} = 9 \text{ km}
\]

\[
A = \frac{9^2 \cdot \frac{2}{3}}{2} = \frac{81 \cdot \frac{2}{3}}{2} = \frac{54}{2} = 27 \text{ km}^2
\]

15. A wheel spins with a linear speed of $22$ m/s and its radius is $200$ cm. How many revolutions/min must the wheel turn? Take $\pi = \frac{22}{7}$.

\[
v = \omega r \quad \omega = \frac{v}{\frac{2}{3}m} = \frac{22}{\frac{2}{3}m} = \frac{33}{7} \text{ rad/s}
\]

\[
r = 200 \text{ cm} = 2 \text{ m} \quad \frac{1 \text{ rev}}{2\pi \text{ rad}} = \frac{11}{\frac{33}{7}} = \frac{11 \cdot 7}{4} = \frac{77}{4} \text{ rev/s}
\]

\[
n = \frac{60 \text{ s}}{4 \frac{3}{5} \text{ min}} = \frac{4 \cdot 4}{4} = \frac{210 \text{ rev}}{25} = 105 \text{ rev/s}
\]

16. Arc length of $r = 10$ m and $\theta = 25^\circ$

\[
\theta = 25^\circ \cdot \frac{\pi}{180^\circ} = \frac{5\pi}{36}
\]

\[
s = r\theta = 10 \text{ cm} \cdot \frac{5\pi}{36} = \frac{25\pi}{18} \text{ cm}
\]

17. What is $\sin \theta$ exactly if $c = 15$ and $b = 9$?

\[
c^2 - b^2 = a^2 \Rightarrow a = \sqrt{225 - 81} = \sqrt{144} = 12
\]

\[
\text{OR} \quad \frac{a}{c} = \sqrt{(\frac{c}{b})^2 - (\frac{b}{c})^2} = \sqrt{5^2 - 3^2} = \sqrt{16} = 4
\]

\[
a = 12
\]

\[
\sin \theta = \frac{12}{15} = \frac{4}{5}
\]