Stresses, Strains, and Deformations

Normal stress due to axial loading: \( \sigma = \frac{P}{A} = \frac{N}{A} \)

Average shear stress: \( \tau_{\text{average}} = \frac{V}{A} \)

Hooke's Law: \( \sigma = E\epsilon \)

Poisson's ratio: \( \nu = -\frac{\varepsilon_{\text{lateral}}}{\varepsilon_{\text{longitudinal}}} \)

Shear strain: \( \gamma = \frac{\pi}{2} - \theta' \) and \( \gamma = \frac{\tau}{G} \)

Elongation for constant normal strain:
\[
\delta = L_f - L_0 = \epsilon L_0
\]

Normal strain due thermal effects: \( \epsilon_{\Delta T} = \alpha \Delta T \)

Total normal strain: \( \epsilon = \frac{\sigma}{E} + \alpha \Delta T \)

Elongation of a prismatic bar:
\[
\delta = L_f - L_0 = \epsilon L_0 = \frac{PL_0}{EA_0} + \alpha \Delta T L_0
\]

Factor of Safety
\[
F.S. = \frac{\sigma_{\text{fail}}}{\sigma_{\text{allow}}}; \quad \text{or} \quad F.S. = \frac{\tau_{\text{fail}}}{\tau_{\text{allow}}}
\]
Alternate forms
\[
F.S. = \frac{F_{\text{fail}}}{F_{\text{allow}}}; \quad F.S. = \frac{\sigma_{\text{allowable}}}{\sigma_{\text{actual}}}
\]

Units and Conversion Factors

Length
- \( m = 1000 \) mm
- \( \text{ft} = 12 \) in
- \( 1 \) inch = 25.4 mm

Force
- \( \text{kN} = 1000 \) N
- \( \text{kip} = 1000 \) lbf.
- \( \text{Newton} = \text{N} = (\text{kg}) \) m / s²
- \( 1 \) N = 0.2248 lb.

Weight
- \( W = (\text{mass})(\text{g}) \)
- \( g = 9.81 \) m / s² or 32.2 ft / s²

Stress
- \( \text{Pa} = \text{N/m}^2 \)
- \( \text{MPa} = 10^6 \) Pa = N/mm²
- \( \text{GPa}=10^9 \) Pa = kN/mm²
- \( \text{psi} = \text{lb.}/\text{in}^2 \)
- \( \text{ksi} = \text{kip}/\text{in}^2 \)

Strain
- \( \mu = \mu \epsilon = 10^{-6} \) in/in = 10⁻⁶ mm/mm
- \( \mu \text{rad} = 10^{-6} \) rad

Trigonometry

\( \sin \theta = \text{Opposite side} / \text{Hypotenuse} \)

\( \cos \theta = \text{Adjacent side} / \text{Hypotenuse} \)

\( \tan \theta = \text{Opposite side} / \text{Adjacent side} \)

Law of Cosines
\[
c^2 = a^2 + b^2 - 2ab \cos \theta
\]

Small Angle Approximation
For \( \beta \) small, \( \beta \approx \sin \beta \approx \tan \beta \); \( \cos \beta \approx 1 \)
1. Member $BC$ has a rectangular cross-section with a 1 1/2” width (as shown) and 1/2” thickness. Pins $A$ and $B$ have 1” diameter. From the cross-section views of the pin joints, we can see that pin $A$ is in single shear while pin $B$ is in double-shear. The allowable normal stress for member $BC$ is 30 ksi. The allowable shear stress for the pins is 12 ksi.

(a) Determine the maximum allowable value of the distributed load $w$ in units of kip/ft such that pins $A$ and $B$ will not fail due to shear stress and member $BC$ will not fail due to normal stress.

(b) For the value of $w$ determined in part (a), determine the values of normal stress in member $BC$, shear stress on pin $A$, and shear stress on pin $B$. 
2. When no load is applied, the original length of the rod is 24.00 inches, and the diameter is 2 inches. The material has the stress-strain curve shown, for which several data points are listed. When an axial tensile force $P$ is applied to the two rigid caps at the ends, the rod deforms in the linear elastic range of the material. The new length becomes 24.03 inches, and the circumferential strain (lateral strain) is $-441 \times 10^{-6}$ in./in., as measured by a strain gage located on the surface of the rod at section $\alpha$.

(a) Determine Young’s Modulus and Poisson’s ratio for the material.

(b) Determine the magnitude of the force $P$, the average normal stress in the rod, and the factor of safety with respect to yield strength.
3. The rigid block has a weight of 80 kip and is to be supported by posts $A$ and $B$, which are made of A-36 steel, and the post $C$, which is made of C83400 red brass. If all the posts have the same original length before they are loaded, determine the average normal stress developed in each post when post $C$ is heated so that its temperature is increased by 20°F. Each post has a cross-sectional area of 8 in$^2$.

C: $E = 14.6 \times 10^6$ psi, $\alpha = 9.8 \times 10^{-6} \frac{1}{\text{oF}}$

A,D: $E = 29 \times 10^6$ psi
4. The 2014-T6 aluminum rod $AC$ is reinforced with the firmly bonded A992 steel tube $BC$. When no load is applied to the assembly, the gap between end $C$ and the rigid support is 0.5 mm. Determine the support reactions when the axial force of 400 kN is applied.

St: $E = 200\times10^9$ Pa

Al: $E = 73.1\times10^9$ Pa