Mechanics of Materials

**Stresses, Strains, and Deformations**

Normal stress due to axial loading: \( \sigma = \frac{P}{A} = \frac{N}{A} \)

Direct average shear stress: \( \tau_{avg} = \frac{V}{A} \)

Hooke’s Law: \( \sigma = E\varepsilon \)

Poisson’s ratio: \( \nu = -\frac{\varepsilon_{lateral}}{\varepsilon_{longitudinal}} \)

Shear strain: \( \gamma = \frac{\pi}{2} - \theta' \); \( \gamma = \frac{\tau}{G} \)

Elongation for constant normal strain: \( \delta = L_f - L_0 = \varepsilon L_0 \)

Normal strain due thermal effects: \( \varepsilon_{\Delta T} = \alpha \Delta T \)

Total normal strain: \( \varepsilon = \frac{\sigma}{E} + \alpha \Delta T \)

Elongation of a prismatic bar:
\[
\delta = L_f - L_0 = \varepsilon L_0 = \frac{PL_0}{EA_0} + \alpha \Delta T L_0
\]

**Torsion, Power Transmission & Gears**

Shear stress due to torsion: \( \tau = \frac{T \rho}{J} \)

Angle of twist: \( \theta = \frac{TL}{JG} \)

Power: \( P = T\omega = T2\pi f \)

Gear ratio: \( \frac{r_1}{r_2} = \frac{N_1}{N_2} = -\frac{T_1}{T_2} = -\frac{\omega_2}{\omega_1} = -\frac{\theta_2}{\theta_1} \)

**Flexure and Shear Flow**

Normal stress due to bending: \( \sigma(y) = -\frac{My}{l} \)

Shear stress due to bending: \( \tau(y) = \frac{VQ}{It} \)

**Factor of Safety**

\[
F.S. = \frac{\sigma_{\text{allow}}}{\sigma_{\text{allow}}}; \text{ or } F.S. = \frac{\tau_{\text{allow}}}{\tau_{\text{allow}}}
\]

Alternate forms
\[
F.S. = \frac{F_{\text{allow}}}{F_{\text{allow}}}; F.S. = \frac{\sigma_{\text{allowable}}}{\sigma_{\text{actual}}}
\]

**Law of Cosines**

\[
c^2 = a^2 + b^2 - 2ab \cos \theta
\]

Exam 2

**Equations sheet**

**Section Properties**

Location of centroid: \( \bar{y} = \frac{\sum y_i A_i}{\sum A_i}; \bar{z} = \frac{\sum z_i A_i}{\sum A_i} \)

Second moment of area: \( I_{\text{comp}} = \sum (I + d_i^2 A_i) \)

First moment of area: \( Q = \bar{y}'A' = \sum y_i'A_i' \)

Rectangular cross-section: \( I = \frac{bh^3}{12}; Q = \frac{bh^2}{8} \)

Circular cross-section:
\[
I = \frac{\pi d^4}{64}; J = \frac{\pi d^4}{32}; Q = \frac{d^3}{12}
\]

Hollow circular cross-section:
\[
I = \frac{\pi(d^4 - d_i^4)}{64}; J = \frac{\pi(d^4 - d_i^4)}{32}; Q = \frac{d^3 - d_i^3}{12}
\]

**Units and Conversion Factors**

Length
- \( m = 1000 \text{ mm} \)
- \( \text{ft} = 12 \text{ inch}; \ 1 \text{ inch} = 25.4 \text{ mm} \)

Force
- \( \text{kN} = 1000 \text{ N} \)
- \( \text{kip} = 1000 \text{ lb} \)
- \( 1 \text{ N} = 0.2248 \text{ lb} \)
- \( \text{Weight} = \text{mass (g)} \)
- \( g = 9.81 \text{ m} / \text{s}^2 \text{ or } 32.2 \text{ ft} / \text{s}^2 \)

Stress
- \( \text{Pa} = \text{N/m}^2 \)
- \( \text{MPa} = 10^6 \text{ Pa} = \text{N/mm}^2 \)
- \( \text{GPa} = 10^9 \text{ Pa} = \text{kN/mm}^2 \)
- \( \text{psi} = \text{lb./in}^2 \)
- \( \text{ksi} = \text{kip/in}^2 \)
- Newton = \( N = (\text{kg}) \text{ m} / \text{s}^2 \)

Strain
- \( \mu = \mu \varepsilon = 10^{-6} \text{ in/ln} = 10^{-6} \text{ mm/mm} \)
- \( \mu \text{rad} = 10^{-6} \text{ rad} \)

Angles
- \( \pi \text{ rad} = 180^\circ \)

Power
- \( \text{hp} = 550 \text{ lb-ft/s} = 745.7 \text{ Watts} \)
- \( \text{Watt} = \text{N-m/s} \)

**Small Angle Approximation**
For \( \beta \) small, \( \beta \sim \sin \beta \sim \tan \beta \); \( \cos \beta \sim 1 \)
The two 3-ft-long shafts are made of 2014-16 aluminum. Each has a diameter of 1.5 in. and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at $A$ and $B$. They are also supported by bearings at $C$ and $D$, which allow free rotation of the shafts along their axes. If a torque of 600 lb·ft is applied to the top gear as shown, determine the maximum shear stress in each shaft.
2) The shaft is used to transmit 30 hp while turning at 600 rpm. Determine the maximum shear stress in the shaft. The segments are connected together using a fillet weld having a radius of 0.18 in.