2. [15 points] A solid 60-mm-diameter cold-rolled brass [G = 39 GPa] shaft that is 1.25-m long extends through and is completely bonded to a hollow aluminum [G = 28 GPa] tube, as shown. Aluminum tube (1) has an outside diameter of 90 mm, an inside diameter of 60 mm, and a length of 0.75 m. Both the brass shaft and the aluminum tube are securely attached to the wall support at A. When the two torques shown are applied to the composite shaft, determine:

(a) (8 pts) The maximum shear stress magnitude in aluminum tube (1).
(b) (3 pts) The maximum shear stress magnitude in brass shaft segment (3).
(c) (4 pts) The rotation angle (in degrees) of a section located 1 meter from point A.

Segment AB is statically indeterminate due to two metals with differing modulus of rigidity (G) compatibility of bonded shaft and tube up to section at B.

$\phi_1 = \phi_2$

$\frac{T_1 L_1}{J_1 G_1} = \frac{T_2 L_2}{J_2 G_2}$

Note $L_1 = L_2 = 750 \text{ mm}$
Work space for Problem 2

Section Properties

\[ J_1 = \frac{\pi}{32} [90^4 - 60^4] = 5.1689 \cdot 10^6 \text{ mm}^4 \quad J_2 = J_3 = \frac{\pi}{32} 60^4 = 1.2724 \cdot 10^6 \text{ mm}^4 \]

Torque-Twist Angle Relationship

\[ T_1 = \frac{J_1 G_1}{J_2 G_2} T_2 = \frac{5.1689 \cdot 10^6}{1.2724 \cdot 10^6} \frac{28}{39} T_2 = (2.9167) T_2 \]

Combine with Equilibrium

\[ 20 - 8 = 12 \text{ kN} \cdot \text{m} = T_1 + T_2 = [2.9167 + 1] T_2 \]

So \[ T_2 = 3.064 \text{ kN} \cdot \text{m} \quad T_1 = 12 - T_2 = 8.936 \text{ kN} \cdot \text{m} \]

a) \[ \tau_1 = \frac{T_1 c_1}{J_1} = \frac{8.936 \cdot 10^6 \text{ N} \cdot \text{mm} (90/2 \text{ mm})}{5.1689 \cdot 10^6 \text{ mm}^4} = 77.8 \frac{\text{N}}{\text{mm}^2} \]

b) \[ \tau_3 = \frac{T_3 c_3}{J_3} = \frac{-8 \cdot 10^6 \text{ N} \cdot \text{mm} (60/2 \text{ mm})}{1.2724 \cdot 10^6 \text{ mm}^4} = 188.6 \frac{\text{N}}{\text{mm}^2} \text{ magnitude} \]

c) Angle of Twist

\[ \phi_{lm/A} = \phi_{lm/B} + \phi_{B/A} = \frac{T_3 (250 \text{ mm})}{J_3 G_3} + \frac{T_2 L_2}{J_2 G_2} \]

\[ = \frac{-8 \cdot (10^6) (250) + 3.064 (10^6) (750)}{1.2724 (10^6) [39,000]} \]

\[ = 0.006005 \text{ rad} \]

\[ = 6.005(10^{-3}) \frac{180}{\pi} = 0.344^\circ \]