1. [14 pts overall] An 8-pole electric motor delivers 10 hp of mechanical power through its armature shaft spinning at nominally 900 revolutions per minute (rpm) into the gear train shown at the right. Spur gear C has a shaft to tooth radius of 30 cm. Gear D has a shaft to tooth radius of 6 cm. The rest of the machine (not shown) driven by gear A uses 4 hp and the machine driven by gear B uses 6 hp.

(a) (3 pts) Determine the torque in the shaft connected to gear D.

(b) (5 pts) The shaft between gears B and C is solid steel. If the steel's allowable shear stress ($\tau_{\text{allowable}}$) is 85 MPa, what is the minimum permitted diameter ($D_2$) of this shaft?

(c) (6 pts) The shaft between gears A and B is a hollow aluminum tube with an outer diameter of 100 mm and an inner diameter of 90 mm. Aluminum's modulus of rigidity (G) is 26 GPa. If the angle of twist between gears A and B must not exceed 1°, determine the maximum permitted length $L_1$.

\[
\omega = \frac{900 \text{ rev/min} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{2\pi \text{ rad}}{\text{rev}}}{5} = 30\pi \frac{\text{rad}}{s}
\]

\[
\omega = 94.25 \frac{\text{rad}}{s}
\]

![Figure P6-22](image)

(a) $\tau = 79.1 \text{ Nm}$

(b) minimum $D_2 = 28.7 \text{ mm}$

(c) maximum $L_1 = 9.68 \text{ m}$

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**a)** Power = 10 hp = $7457 \text{ W} = 7457 \text{ Nm/s}$

\[
T_D = \frac{\text{Power}}{\omega} = \frac{7457 \text{ Nm/s}}{94.25 \text{ rad/s}} = 79.12 \text{ Nm}
\]

**b)**

\[
T_c = \frac{r_c}{r_D} T_D = \frac{30 \text{ cm}}{6 \text{ cm}} \times 79.12 \text{ Nm} = 395.6 \text{ Nm}
\]

\[
\tau_{\text{allow}} = 85 \frac{\text{N}}{\text{mm}^2} = \frac{\tau}{G} = \frac{T_c (D_2/2)}{\frac{\pi}{32} (D_2)^4} = \frac{16 (395.600 \text{ N mm})}{\pi (D_2)^3}
\]

**c)**

\[
T_A = 0.4 T_c = 158,240 \text{ N mm} \quad 1^\circ = 0.01745 \text{ rad}
\]

\[
J_{A1} = \frac{\pi}{32} \left[ (100 \text{ mm})^4 - (90 \text{ mm})^4 \right] = 3.376 \times 10^6 \text{ mm}^4
\]

\[
\phi_{A/B} = \frac{T_A L_1}{J_{A1} G} \quad 0.01745 = \frac{0.15824 \times 10^6 \text{ N mm} L_1}{3.376 \times 10^6 \text{ mm}^4 \times 26 \times (10)^3 \text{ N/mm}^2}
\]