Show all of your work and circle your section number. Final answers must be shown in the space provided on this page.

3. [15 pts] A fabricated wooden I-beam is constructed by gluing three rectangular boards and has cross-sectional dimensions as shown at the right. This beam supports a uniformly distributed load of 2 kN/m and is simply supported as depicted below.

(a) (2 pts) Draw a free-body diagram of the beam beginning at the left end and ending on a cut section at \( X = 1 \) meter. (2 pts) Determine the internal loads acting on the cut section.

(b) (1 pts) Determine the location \( x \) where the bending moment is greatest.

(c) (3 pts) Determine the largest magnitude compressive stress in the fabricated wooden I-beam.

(d) (3 pts) Determine the maximum shear stress at the location where \( X = 1 \) m.

(e) (1 pts) Write an equation for the shear force in the beam as a function of \( X \).

(f) (3 pts) Determine the minimum necessary adhesive strength of the glue which bonds the three board together.

\[ 2 \text{ kN/m} \]

INCLUDE UNITS for all answers

(a) \( V(x=1) = -2 \text{ kN down} \)

(b) \( M(x=1) = +3 \text{ kN m} \)

(c) \( \sigma = 4.006 \text{ MPa} \)

(d) \( \tau_{\text{wood max}} = 0.325 \text{ MPa} \)

(e) \( V(x) = [4 - 2x] \text{ kN} \)

(f) \( \tau_{\text{glue min}} = 0.577 \text{ MPa} \)

Statics on beam

\[ F_L + F_R = (4 \text{ m})(2 \text{ kN/m}) = 8 \text{ kN} \]

\[ (4 \text{ m}) F_L = (2 \text{ m}) 8 \text{ kN} = 0 \]

mean \( F_L = F_R = 4 \text{ kN} \)

Statics on segment

\[ 4 \text{ kN} + V - (1 \text{ m})(2 \text{ kN/m}) = 0 \]

\[ 2 \text{ kN (0.5 m)} + M - (4 \text{ kN})(1) = 0 \]

mean \( V = -2 \text{ kN} \)

\[ M = +3 \]
(c) Maximum moment occurs at mid-span 
\[ (4\, \text{kN})(2\, \text{m}) - (2\, \text{kN/m})(2\, \text{m})(1\, \text{m}) = 4\, \text{kNm} \]

Maximum compressive stress occurs at the top surface of I-beam with (+) moment 
\[ c = 100\, \text{mm} \]

2nd Area Moment 
\[ I_{NA} = \frac{1}{12} \left[ (180\, \text{mm})(200\, \text{mm})^3 - (180-40\, \text{mm})(120\, \text{mm})^3 \right] \]
\[ = 99.84\, (10)^6\, \text{mm}^4 \]

\[ \sigma = \frac{M_c}{I} = \frac{4(10)^6\, \text{Nmm}(100\, \text{mm})}{99.84\, (10)^6\, \text{mm}^4} = \frac{4.006\, \text{N}}{\text{mm}^2} = 4.01\, \text{MPa} \]

(d) Maximum Shear Stress occurs at Neutral Axis 
\[ Q_{NA} = \sum \bar{Y}'a' = 30\left[(40)(60) + (60+20)(180)(40)\right] \]
\[ = 648\, (10)^3\, \text{mm}^3 \]
\[ \tau_{\text{wood max}}(x=1\, \text{m}) = \frac{VQ_{NA}}{I + t} = \frac{(2000\, \text{N})(648\, (10)^3\, \text{mm}^3)}{99.84\, (10)^6\, \text{mm}^4(40\, \text{mm})} \]
\[ = 0.3245\, \text{N/mm}^2 = 0.325\, \text{MPa} \]

(f) Maximum shear stress in glue bond occurs at maximum shear force location (left and right ends) 
\[ Q_{\text{glue}} = \bar{Y}'a' = (60+20)\, \text{mm} \left[ (180)(40)\, \text{mm}^2 \right] \]
\[ = 576\, (10)^3\, \text{mm}^3 \]
\[ \tau_{\text{glue min}} = \frac{V_{\text{max}}Q_{\text{glue}}}{I + t} = \frac{(4000\, \text{N})(576\, (10)^3\, \text{mm}^3)}{99.84\, (10)^6\, \text{mm}^4(40\, \text{mm})} \]
\[ = 0.576\, \text{N/mm}^2 = 0.577\, \text{MPa} \]