1. [20 pts] An elastic plate (PQRS) is initially in a perfect rectangular shape. Loads are applied which alter this shape to the dashed outline below.
   a. Determine the normal strain along the deformed line P’ S’.
   b. Determine the shear strain ($\gamma_{xy}$) at the corner “P”.

\[ \psi_1 = \arctan \left( \frac{9 - 2}{910} \right) \approx \frac{7}{910} \]

increases the included angle at point P

\[ \psi_2 = \arctan \left( \frac{3}{502} \right) \approx \frac{3}{502} \]

but decreases the included angle at point P

Recall shear strain sign convention for angle changes

\[ \gamma_{xy} = \psi_1 - \psi_2 \approx \frac{7}{910} - \frac{3}{502} \approx 0.00172 \text{ rad} \]

\[ \text{INCLUDE customary units} \]

\[ \begin{align*}
  \text{a) } & \quad 0.0111 \frac{\text{mm}}{\text{mm}} \\
  \text{b) } & \quad 0.00172 \text{ rad}
\end{align*} \]
2. [45 pts] A round bar has a flat oversize end with a hole for a 60 mm diameter pin. See Section a-a diagram below for other dimensions of the oversize end. This rod is connected to a wall on the left with a clevis bracket. The clevis then puts the pin into a “double shear” condition. Assume the clevis bracket is much stronger than the pin or rod. The stress-strain diagram of the bar material is given below, and its Poisson’s ratio is 0.27.

a. If the bearing stress limit is 12 MPa, determine the maximum load $P$ that may be applied.
b. If the load “$P$” is 30 kN and the pin will fail in shear at a stress of 15 MPa, determine the factor of safety of the pin.
c. Determine the longitudinal normal strain $\varepsilon$ in the bar, for load “$P$” equal 20 kN.
d. Determine the CHANGE in diameter of the round part of the bar, after the load of part c). is applied.
e. If the bar material is loaded to a stress of 250 MPa, determine the resulting permanent offset.
f. Determine the Factor of Safety with respect to engineering fracture strength at locations in the bar far away from the oversized end, when “$P$” = 30 kN.

INCLUDE units when appropriate:

a) $P =$ \underline{36 \text{ kN}}
b) F. S. = \underline{2.83}
c) $\varepsilon =$ \underline{239 \text{ mm/mm}}
d) $\Delta d =$ \underline{-2.58 \left(10^{-6}\right) \text{ m}}
e) \underline{0.00125 \text{ or } 0.125 \%}
f) F. S. = \underline{12.6}

\[ E = \text{slope} = \frac{\Delta \sigma}{\Delta \varepsilon} = \left[ \frac{200-0}{0.003-0} \right] \text{MPa} = 66,667 \text{ MPa} \]
Show your work; Final answers including units must be shown in the boxes.

Work space for problem 2

a) \[ P_{\text{max}} = \bar{F}_{\text{allow}} A_{\text{projected}} = \left[ 12 \frac{N}{mm^2} \right] \left[ (50 \text{mm})(60 \text{mm}) \right] = 36 \times 10^3 \text{N} \]

b) \[ F.S. = \frac{\bar{F}_{\text{fail}}}{\bar{F}_{\text{allow}}} = \frac{15 \text{ MPa}}{\frac{30 \text{ kN}}{2 \pi (30 \text{mm})^2}} = 2.827 \]

c) \[ \bar{\sigma}_{\text{bar}} = \frac{\bar{F}}{A} = \frac{20 \times 10^3 \text{ N}}{\pi (20 \text{mm})^2} = 15.915 \frac{N}{mm^2} = 15.915 \text{ MPa} \]

\[ \varepsilon_{\text{long}} = \frac{\bar{\sigma}}{E} = \frac{15.915}{66,667} = 0.0002387 = 2.39 \times 10^{-6} \frac{\text{mm}}{\text{mm}} \]

d) \[ \varepsilon_{\text{lat}} = -\nu \varepsilon_{\text{long}} = -(0.27) [238.7 \mu] = -64.46 \mu \]

\[ \Delta d = \varepsilon_{\text{lat}} L_0 = -64.46 \times 10^{-6} [40 \text{mm}] = -0.002578 \text{mm} \]

e) Strain at Stress = 250 MPa is 0.005

Elastic recovery line on a stress-strain diagram is parallel to initial elastic deformation. "Parallel" implies the same slope.

By similar triangles

Permanent Offset = 0.005 - \frac{250}{200} = 0.003 = 0.00125

f) \[ F.S. = \frac{S_{\text{frac}}}{\bar{\sigma}_{\text{act}}} = \frac{300 \text{ MPa}}{\frac{30 \times 10^3 \text{ N}}{\pi (20 \text{mm})^2}} = 12.57 \]
3. [35 pts] A cylindrical magnesium bar (1) is assembled between two rigid plates (2) using two stainless steel carriage bolts (3) as shown. The steel \( E = 195 \text{ GPa}; \alpha = 17 \times 10^{-6}/\text{°C} \) bolts have a diameter of 12 mm. The magnesium \( E = 45 \text{ GPa}; \alpha = 26 \times 10^{-6}/\text{°C} \) bar has a diameter of 40 mm and a length of 100 mm. The initial temperature of all components is \( T_i = 15 \text{ °C} \) prior to assembly. Assume the plates have an alpha value equal to zero; and no weight.

a. Prior to the bolts and nuts being installed, determine the elongation of the bar (due only to the temperature change), if the bar were heated to \( T_f = 110 \text{ °C} \).

The initial temperature is reset to \( T_i = 15 \text{ °C} \), and the nuts are then hand-tightened on the bolt until the bolts, plates, and bar are just snug (meaning that no tensile or compressive stresses are created in the bar or bolts, but no gaps exist either). If the assembly is heated to \( T_f = 110 \text{ °C} \), calculate:

b. The normal stress \( \sigma_{\text{bolt}} \) in the bolts.

b. The total normal strain \( \varepsilon_{\text{bar}} \) in the magnesium bar.

\[
\delta = \frac{0.247 \text{ mm}}{} \\
\sigma_{\text{bolt}} = 4.06 \text{ MPa [T]} \\
\varepsilon_{\text{bar}} = \frac{0.00245 \text{ mm}}{} \\
\]

\[
a) (S_{\text{bar}})_T = \alpha \Delta T L \\
\Delta T = 110 - 15 = 95 \text{ °C} \\
\alpha = 26 (10^{-6}) /\text{°C} \\
(S_{\text{bar}})_T = 26 (10^{-6}) [95] [100] \\
= 0.247 \text{ mm} \\
b) \text{ likewise} \\
(S_{\text{bolt}})_T = 17 (10^{-6}) [95] [150 \text{ mm}] = 0.24225 \text{ mm}
\]
Equilibrium from Section Cut through bar and bolts

\[ \uparrow \Sigma F_y = F_{\text{bar}} + 2F_{\text{bolt}} = 0 \]

\[ F_{\text{bolt}} = - \frac{1}{2} F_{\text{bar}} \]

Compatibility Condition

\[ (\sigma_{\text{bar}})_T + (\sigma_{\text{bar}})_F = (\sigma_{\text{bolt}})_T + (\sigma_{\text{bolt}})_F \]

Load Displacement Relationship

\[ (\sigma_{\text{bar}})_F = \frac{PL}{AE} = \frac{F_{\text{bar}} [100 \text{ mm}]}{\nu (20 \text{ mm})^2 [45 (10^3) \text{ N/mm}^2]} \]

\[ (\sigma_{\text{bolt}})_F = \frac{F_{\text{bolt}} [150 \text{ mm}]}{\nu (6 \text{ mm})^2 [195 (10^3) \text{ N/mm}^2]} \]

Combine into Compatibility Condition

\[ 0.2470 \text{ mm} + \left[ 1.76839 \frac{\text{mm}}{N} \right] (10^{-6}) F_{\text{bar}} \]

\[ = 0.24225 \text{ mm} + \left[ 6.80149 \frac{\text{mm}}{N} \right] (10^{-6}) F_{\text{bolt}} \]

\[ \left[ 0.2470 - 0.24225 \right] \text{ mm} = \left[ 6.80149 + 2(1.76839) \right] \frac{\text{mm}}{N} (10^{-6}) F_{\text{bolt}} \]

\[ F_{\text{bolt}} = 459.5 \text{ N} \]

\[ F_{\text{bar}} = -918.9 \text{ N} \]
Work space for problem 3

\[ \sigma_{\text{bolt}} = \frac{F_{\text{bolt}}}{Area} = \frac{459.5 \text{ N}}{\pi (6 \text{ mm})^2} = 4.06 \frac{\text{N}}{\text{mm}^2} \ [\text{T}] \]

c) Total Strain in Bar

\[ \epsilon_{\text{bar}} = \frac{\sigma_{\text{bar}}}{E} + \alpha \Delta T \]

\[ = \frac{-918.9 \text{ N}}{\pi (20 \text{ mm})^2 \left[45 \left(10^3\right) \text{ N/mm}^2\right]} + 26 \times 10^{-6} \times 95 \]

\[ = 0.002454 \]