Show all of your work.

1. A plate with corners A and C on the x-axis is attached to a wall in the y-z plane with a cable at point B and two hinges along its edge CD. The tension in cable AB is \( F = 140 \text{ lb} \) and acts on the plate at point A as shown.

(a) Determine the moment of the tension force about point C. Report your answer in Cartesian vector form.

(b) Determine the magnitude of the moment of the tension force about the hinge line CD.

\[
P + A \ (6, 0, 0) \\
P + B \ (0, 4, 12) \\
P + C \ (0, 8, 6)
\]

\[
\overrightarrow{R_{AB}} = (6-0)\hat{i} + (4-0)\hat{j} + (12-0)\hat{k} = 6\hat{i} + 4\hat{j} + 12\hat{k} \text{ ft}
\]

\[
\overrightarrow{V_{AB}} = \frac{-6\hat{i} + 4\hat{j} + 12\hat{k}}{\sqrt{6^2 + 4^2 + 12^2}} = -0.429\hat{i} + 0.286\hat{j} + 0.857\hat{k}
\]

\[
F = 140 \overrightarrow{V_{AB}} = [-60\hat{i} + 40\hat{j} + 120\hat{k}] \text{ lb}
\]

\[
\overrightarrow{U_{CD}} = \frac{8\hat{j} + 6\hat{k}}{\sqrt{8^2 + 6^2}} = 0.8\hat{j} + 0.6\hat{k}
\]

\[
\|M_{CD}\| = \overrightarrow{U_{CD}} \times (\overrightarrow{V_{CA}} \times \overrightarrow{F})
\]

\[
\overrightarrow{V_{CA}} = [6\hat{i}] \text{ ft}
\]

\[
\overrightarrow{V_{CA}} \times \overrightarrow{F} = \begin{vmatrix}
\hat{i} & \hat{j} & \hat{k} \\
6 & 0 & 0 \\
-60 & 40 & 120
\end{vmatrix} = [-720\hat{j} + 240\hat{k}] \text{ ft-lb}
\]

\[
\|M_{CD}\| = (0.8\hat{j} + 0.6\hat{k}) \times (-720\hat{j} + 240\hat{k})
\]

\[
\|M_{CD}\| = -432 \text{ ft-lb}
\]

(a) \( M_C = -720\hat{j} + 240\hat{k} \text{ ft-lb} \)

(b) \( M_{||CD} = 432 \text{ ft-lb} \)
Show all of your work.

3. The distributed load acts on the beam $AB$ as shown.
   (a) Determine the force-couple system at point $A$ that is equivalent to the original distributed load.
   (b) Replace the original distributed load with an equivalent resultant force, and specify its location measured from point $A$.

\[ F_1 = 6 \times 6 = 36 \text{ kN} \]
\[ F_2 = 2 \times 14 = 28 \text{ kN} \]
\[ F_3 = 6 \times 8 = 48 \text{ kN} \]
\[
\begin{align*}
(M_R)_A &= \sum M_A = -18(4) - 28(7) - 24(8.67) \\
\vec{F}_R &= 1 \sum F_i = -18 - 28 - 24 \\
x &= \frac{476.1}{70} = 6.8 \text{ m}
\end{align*}
\]

\[
\begin{align*}
\vec{F}_R &= \begin{bmatrix} 70 \text{ kN} \end{bmatrix} \\
(M_R)_A &= \begin{bmatrix} 476.1 \text{ kN.m} \end{bmatrix} \\
\vec{F}_R &= \begin{bmatrix} 70 \text{ kN} \end{bmatrix} \\
x &= 6.8 \text{ m}
\end{align*}
\]
Show all of your work and circle your section number

2  3  4  5  6  8  10  11  12  Exam ID

1. [25 pts] Replace applied loads with statically equivalent force-couple system at point C, and present force in Cartesian vector form, and couple in the form of magnitude and direction.

Neglect the self-weight of the beam.

\[ \sum F_y = -600 - 750 - 600 = -1950 \text{ N} \]

\[ \sum M_c = 550 - 150 + 600(2) - 600(1.67) \]

\[ = 598 \text{ Nm} \]

Include UNITS:

\[ \mathbf{F} = [0 \hat{i} -1950 \hat{j}] \text{ N} \]

\[ \mathbf{M}_c = [0 \hat{i} + 0 \hat{j} + 598 \hat{k}] \text{ Nm} \]
2. The weightless pipe assembly is supported in the horizontal plane by a smooth collar at \( A \) and rests on a smooth surface support at \( B \). Two vertical forces are applied as shown.
(a) Draw a free-body diagram of the pipe assembly \( AB \).
(b) Determine the support reactions at \( A \) and \( B \). Report your answers in Cartesian vector form.

You will receive no credit for part (b) if you do not draw the free-body diagram.

\[
\begin{align*}
\mathbf{R}_A &= R_{Ay} \hat{j} + R_{Az} \hat{k} & \sum F &= 0; \quad \hat{i}: F_x &= 0 \\
\mathbf{R}_B &= R_{Bz} \hat{k} & \hat{j}: F_y &= R_{Ay} = 0 \\
\mathbf{M}_A &= M_{Ay} \hat{j} + M_{Az} \hat{k} & \hat{k}: F_z &= R_{Az} + R_{Bz} - 800 - 600 = 0 \\
\sum M_A &= M_A \\
(0.8\hat{i} + 1.6\hat{j} \times -800\hat{k}) + (-0.4\hat{i} + 1.6\hat{j} \times -600\hat{k}) + (-0.8\hat{i} + 1.6\hat{j} \times 800\hat{k}) \\
&= (-640\hat{i} - 240\hat{j} - 960\hat{k} + 0.8R_{Bz}\hat{i} + 1.6R_{Bz}\hat{k}) = 0 \\
\hat{i}: M_{Ay} - 240 + 0.9R_{Bz} &= 0 \\
\hat{k}: M_{Az} &= 0 \\
\end{align*}
\]

1. \( R_{Bz} = \frac{1600}{1.6} = 1000 \) N
2. \( M_{Ay} = -0.3(1000) + 240 = -560 \) Nm

From \( \sum F \):
\(
\begin{align*}
\hat{i}: R_{Ay} &= 0 \\
\hat{k}: R_{Az} + 1000 - 800 - 600 &= 0 \\
\Rightarrow R_{Az} &= 400 \text{ N} \\
\end{align*}
\)

\(
\begin{align*}
\mathbf{R}_A &= [0\hat{i} + 0\hat{j} + 400\hat{k}] \text{ N} \\
\mathbf{R}_B &= [0\hat{i} + 0\hat{j} + 1000\hat{k}] \text{ N} \\
\mathbf{M}_A &= [0\hat{i} + 560\hat{j} + 0\hat{k}] \text{ Nm} \\
\mathbf{M}_B &= 0 \\
\end{align*}
\)

(b) \( \mathbf{F}_A = [0\hat{i} + 0\hat{j} + 400\hat{k}] \text{ N} \)
\( \mathbf{M}_A = [0\hat{i} - 560\hat{j} + 0\hat{k}] \text{ Nm} \)
\( \mathbf{F}_B = [0\hat{i} + 0\hat{j} + 1000\hat{k}] \text{ N} \)
\( \mathbf{M}_B = [0\hat{i} + 0\hat{j} + 0\hat{k}] \text{ Nm} \)
3. [25 pts] The hoist pulley structure is rigidly attached to the wall at C. A load of sand hangs from the cable which passes around 1-ft diameter, frictionless pulley at D. The weight of the sand can be treated as a triangular distributed load with a maximum of 70 lb/ft. Determine all reactions acting on member ABC, and show them on report diagram in the form of magnitude and direction.

Neglect the self-weight of the structure.

Work without FBD will not receive any credit.

\[
\begin{align*}
\text{Distributed Load:} \quad W &= 2 \left( \frac{70 \text{ lb/ft} \times 1.5 \text{ ft}}{2} \right) = 105 \text{ lb} \\
T &= W = 105 \text{ lb}
\end{align*}
\]

\[
\begin{align*}
\Sigma M_A &= 0; \quad -105 (1.5) - 105 (2) + F_{BE} (4) = 0 \\
&\quad -157.5 - 210 + 4F_{BE} = 0 \\
F_{BE} &= 91.78 \text{ N}
\end{align*}
\]

\[
\begin{align*}
\Sigma F_y &= 0; \quad -A_y - 105 + 91.78 = 0 \\
&\quad A_y = -13.12 \text{ N} \\
A_y &= 105 + 91.78
\end{align*}
\]

\[
\begin{align*}
\Sigma F_x &= 0; \quad -A_x + 105 = 0 \\
&\quad A_x = 105 \text{ N}
\end{align*}
\]

\[
\begin{align*}
\Sigma F_y &= 0; \quad C_y - F_{BE} + A_y = 0 \\
&\quad C_y - 91.78 - 13.12 = 0 \\
C_y &= 105 \text{ N}
\end{align*}
\]

\[
\begin{align*}
\Sigma F_x &= 0; \quad -C_x + A_x = 0 \\
&\quad -C_x + 105 = 0 \\
&\quad C_x = 105 \text{ N}
\end{align*}
\]

\[
\begin{align*}
\Sigma M_c &= 0; \quad M_c + 3F_{BE} - 7A_y = 0 \\
M_c + 3(91.78) - 7(-13.12) &= 0 \\
\Rightarrow M_c &= -367.48 \text{ Nm}
\end{align*}
\]

\[
\begin{align*}
M_c &= 275.64 + 91.78 = 0
\end{align*}
\]
4. A force $P$ is applied to the handle of the toggle clamp, resulting in a clamping force $F$ at point $C$. The system is in equilibrium for the orientation shown. Neglect weight.

(a) Draw a free-body diagram of the handle.
(b) Draw a free-body diagram of the shaft $CB$, which slides freely in its smooth guide.
(c) Draw a free-body diagram of link $AB$.
(d) Determine the magnitudes of forces $P$ and $F$.
(e) Determine the magnitude of the pin force at $O$.

You will receive no credit for parts (d) and (e) if you do not draw the free-body diagrams.

FBD a) 
\[
\begin{align*}
\sum F_y &= 0; \quad 0 - F_{AB}\sin 15 - 30\cos 15 &= 0 \\
\sum M_o &= 0; \quad F_{AB}\sin 15 (2\cos 15) + 30\cos 15 (8\cos 15) + F_{AB}\cos 15 (2\sin 15) + 30\sin 15 (8\sin 15) &= 0 \\
F_{AB} (0.5) &= 223.92 + F_{AB} (0.5) + 16.07 = 0 \\
F_{AB} &= -240 N
\end{align*}
\]

FBD b) 
\[
\begin{align*}
\sum F_x &= 0; \quad -30\sin 15 - F_{AB}\cos 15 - 0_x &= 0 \\
0 - F_{AB}\sin 15 - 30\cos 15 &= 0 \\
0 - (-240)\sin 15 - 30\cos 15 &= 0 \\
0_y &= -33.1 N \\
-30\sin 15 - F_{AB}\cos 15 - 0_x &= 0 \\
-30\sin 15 - (240)\cos 15 - 0_x &= 0 \\
0_x &= 224.1 N
\end{align*}
\]

FBD c) 
\[
\begin{align*}
\sum F_x &= 0; \quad F + F_{AB}\cos 15 &= 0 \\
F &= -(-240)\cos 15 \\
F &= 231.8 lb
\end{align*}
\]

\[
O = \sqrt{0_x^2 + 0_y^2} = \sqrt{224.1^2 + 33.1^2} \\
O &= 226.5 lb
\]