PROBLEM 6.43

Air is compressed adiabatically in a piston-cylinder assembly from 1 bar, 300 K to 10 bar, 600 K. The air can be modeled as an ideal gas and kinetic and potential energy effects are negligible. Determine the amount of entropy produced, in kJ/K per kg of air, for the compression. What is the minimum theoretical work input, in kJ per kg of air, for an adiabatic compression from the given initial state to a final pressure of 10 bar?

KNOWN: Air is compressed adiabatically in a piston-cylinder. The initial and final states are specified.

FIND: Determine the amount of entropy produced and the minimum theoretical work input for adiabatic compression from the initial state to the given final pressure, each per unit mass of air.

SCHEMATIC AND GIVEN DATA:

ENGINEERING MODEL: (1) The air is a closed system. (2) $Q = 0$ and kinetic and potential energy effects are negligible. (3) The air is modeled as an ideal gas.

ANALYSIS: To find the entropy produced, begin with the entropy balance: $\Delta S = \int_1^T \left( \frac{dQ}{T} \right) + \sigma$

Thus

$\sigma/m = s_2 - s_1 = s^0(T_2) - s^0(T_1) - R \ln(p_2/p_1)$

With data from Table A-22

$\sigma/m = (2.40902 - 1.70203) \text{kJ/kg} \cdot \text{K} - \left( \frac{8.314}{28.97} \text{kJ/kg} \cdot \text{K} \right) \ln\left( \frac{10 \text{ bar}}{1 \text{ bar}} \right) = 0.04618 \text{ kJ/kg} \cdot \text{K}$

The work is determined using the energy balance, as follows: $\Delta KE + \Delta PE + \Delta U = Q - W$

$W/m = (u_1 - u_2)$

As $u$ varies with $T$ for an ideal gas, the work input decreases as $T_2$ decreases. From the entropy balance

$s_2 - s_1 = \sigma/m \geq 0 \rightarrow s_2 \geq s_1$
As shown on the accompanying T-s diagram, only states at 10 bar to the right of state 2s (isentropic compression – with $\sigma/m = 0$) are accessible in an adiabatic compression, and the corresponding temperature $T_{2s}$ is the lowest possible temperature. Hence, compression to state 2s gives the minimum theoretical work input.

For $s_2 - s_1$: $s^0(T_{2s}) = s^0(T_1) + R \ln(p_2/p_1) = 1.70203 + (8.314/28.97) \ln (10/1) = 2.36284 \text{ kJ/kgK}$

Interpolating in Table A-22: $T_{2s} \approx 564.1 \text{ K}$ and $u_2 \approx 407.55 \text{ kJ/kg}$. With $u_1 = 214.07 \text{ kJ/kg}$

$$(W/m)_{\text{min input}} = (u_{2s} - u_1) = 407.55 - 214.07 = 193.48 \text{ kJ/kg}$$

1. Note: The work for actual process from state 1 to state 2 is

$$(W/m)_{\text{input}} = (u_2 - u_1) = 434.78 - 214.07 = 220.71 \text{ kJ/kg}$$

which is greater than the theoretical minimum, as expected.
PROBLEM 6.120

Steam undergoes an isentropic compression in an insulated piton-cylinder assembly from an initial state where \( T_1 = 120^\circ\text{C}, p_1 = 1 \text{ bar} \) to a final state where the pressure is \( p_2 = 100 \text{ bar} \). Determine the final temperature, in \(^\circ\text{C}\), and the work, in kJ per kg of steam.

KNOWN: Steam undergoes an isentropic compression in an insulated piston-cylinder assembly. The initial state is fixed and the final pressure is specified.

FIND: Determine the final temperature and the work per unit mass of steam.

SCHEMATIC AND GIVEN DATA:

![Diagram of steam cycle]

\( T_1 = 120^\circ\text{C} \)
\( p_1 = 1 \text{ bar} \)
\( p_2 = 100 \text{ bar} \)
\( s_2 = s_1 \)

ENGINEERING MODEL: (1) The steam is a closed system. (2) \( Q = 0 \) and kinetic and potential energy effects can be neglected. (3) The process is internally reversible, and \( s_2 = s_1 \).

ANALYSIS: To fix state 2, we use the pressure, 100 bar, and the specific entropy: \( s_2 = s_1 \). From Table A-4, at \( p_1 = 1 \text{ bar} \), \( T_1 = 120^\circ\text{F}; s_1 = 7.4668 \text{ kJ/kg} \cdot \text{K} \). Also, \( u_1 = 2537.3 \text{ kJ/kg} \).

The highest specific entropy value in Table A-4 at 100 bar is 7.2670 kJ/kg·K (at 740\( ^\circ\text{C} \)). Extrapolating ; \( T_2 \approx 821.3^\circ\text{C} \). Further, \( u_2 \approx 3669.4 \text{ kJ/kg} \).

Using \( IT \) the values are \( T_2 = 826.1^\circ\text{C} \) and \( u_2 = 3680 \text{ kJ/kg} \). These values are more accurate, and will be used for further calculations.

The work is obtained using the closed system energy balance, which reduces as follows:

\[
\Delta KE + \Delta PE + \Delta U = Q - W \quad \rightarrow \quad W/m = (u_1 - u_2)
\]

or

\[
W/m = 2537.3 - 3680 = -1142.7 \text{ kJ/kg}
\]
Air contained in a rigid, insulated tank fitted with a paddle wheel, initially at 1 bar, 330 K and a volume of 1.93 m³, receives an energy transfer by work from the paddle wheel in an amount of 400 kJ. Assuming the ideal gas model for the air, determine (a) the final temperature, in K, (b) the final pressure, in bar, and (c) the amount of entropy produced, in kJ/K. Ignore kinetic and potential energy.

**Known:** Air in a rigid, insulated tank is stirred by a paddle wheel. State data and W are given.

**Find:** Determine the final temperature and pressure and $\Delta S$.

**Engineering Model:**
1. The air in the tank is the closed system.
2. For the system, $Q = 0$ and there are no effects of kinetic and potential energy.
3. The air is modeled as an ideal gas.

**Analysis:**
(a) Applying an energy balance, $\Delta U + \Delta PE + \Delta KE = Q - W$

$b$ $\Rightarrow \Delta U = -W$. Or, $m(u_2 - u_1) = -W \Rightarrow u_2 = u_1 - \frac{W}{m}$ (1)

We get $u_1$ from Table A-22: $u_1 = 235.61 \text{ kJ/kg}$. To obtain $m$, apply the ideal gas model equation of state:

$$m = \frac{P_1 V}{RT_1} = \frac{(10^5 \text{ N/m}^2)(1.93 \text{ m}^3)}{(8314 \text{ N} \cdot \text{m/kg} \cdot \text{K})(330 \text{ K})} = 204 \text{ kg}$$

Then Eq. (1) gives

$$u_2 = \left[235.61 - \frac{-400}{2.04}\right] \text{kJ} = 431.69 \text{ kJ/kg}$$

Interpolating with Table A-22, $T_2 = 596 \text{ K}$

(b) Applying the ideal gas model equation of state,

$$\frac{P_1 V}{mRT_1} \Rightarrow \frac{P_2}{P_1} = \frac{T_2}{T_1} \Rightarrow P_2 = P_1 \left[\frac{T_2}{T_1}\right] = 1 \text{ bar} \left[\frac{596 \text{ K}}{330 \text{ K}}\right] = 1.81 \text{ bar}$$

(c) Applying an entropy balance, $\Delta S = \int \frac{S_2 - S_1}{T_1}$

$\Rightarrow \Delta S = m[S_2 - S_1] = m \left[\frac{596}{28.97} - R \ln \frac{P_2}{P_1}\right]$.

With $S_0$ data from Table A-22,

$$\Delta S = 2.04 \text{ kJ} \left[2.81088 - 1.79788 - \frac{8.314}{28.97} \ln (1.81)\right] \frac{\text{kJ}}{\text{K}}$$

$$= 0.885 \text{ kJ/K}$$