Useful Equations

\[ \overline{B} = \frac{\mu_0}{4\pi} \frac{qv \times \hat{r}}{r^2} \]

Magnetic field of a point charge with constant velocity

\[ d\overline{B} = \frac{\mu_0}{4\pi} \frac{I d\hat{l} \times \hat{r}}{r^2} \]

Magnetic field of a current element

\[ B = \frac{\mu_0 I}{2\pi r} \]

Magnetic field near a long, straight, current-carrying conductor

\[ \frac{F}{L} = \frac{\mu_0 I I'}{2\pi r} \]

Force between two long, parallel, current-carrying conductors

\[ B_x = \frac{\mu_0 I a^2}{2 \left( x^2 + a^2 \right)^{3/2}} \]

Magnetic field along axis of symmetry for a current-carrying loop

\[ B_y = \frac{\mu_0 N I}{2a} \]

Magnetic field at the center of N circular loops

\[ \mu_0 = 4\pi (10^{-7}) \text{ Tm/A} \]

Related Problems

1) An electron and a proton are each moving at 850 km/s in perpendicular paths as shown in the figure. At the instant they are at the positions shown in the figure. (Book 28.8)

(a) Find the magnitude and direction of the total magnetic field they produce at the origin.

\[ B_p = \frac{\mu_0 + q_e (-v) \sin(90)}{4\pi} \frac{4 (10^{-9})}{5 (10^{-9})} \]

\[ B_e = \frac{\mu_0 - q_e v \sin(90)}{4\pi} \frac{4 (10^{-9})}{5 (10^{-9})} \]

\[ B_z = \frac{\mu_0 q v}{4\pi} \left( -\frac{1}{4 (10^{-9})} - \frac{1}{5 (10^{-9})} \right) = -1.39 \text{ T} \]

(Negative means it is going into the page)

(b) Find the magnitude and direction of the magnetic field the electron produces at the location of the proton.
\[ B_x = \frac{\mu_0}{4\pi} \frac{-q_v \sin\left(\frac{\pi}{1.3986}\right)}{\sqrt{[4(10^{-9})]^2 + [5(10^{-9})]^2}} = -2.59 \times 10^{-4} \text{ T} \]

(Notice that the angle between the velocity vector and \(r\)-hat)

(c) Find the magnitude of the total magnetic force and the total electrical force that the electron exerts on the proton.

\[ F_{\text{electric}} = \frac{kq_e^2}{r^2} = \frac{kq_e^2}{(\sqrt{41(10^{-9})})^2} = 5.62 \times 10^{-12} \text{ N} \]

\[ F_{\text{magnetic}} = qv \times B = qvB \sin(\phi) = qv \frac{\mu_0}{4\pi} \frac{-q_v \sin\left(\frac{\pi}{1.3986}\right)}{\sqrt{[4(10^{-9})]^2 + [5(10^{-9})]^2}} = 3.52 \times 10^{-17} \text{ N} \]

2) Two parallel wires are 5.20 cm apart and carry currents in opposite directions, as shown in the figure. Find the magnitude of the magnetic field at point P due to two 1.50 mm segments of wire that are opposite each other and each 8.00 cm from P. (Book 28.12)

\[ \frac{dB}{dr} = \frac{\mu_0}{4\pi} \frac{I \partial \times \hat{r}}{r^2} \]

\[ dl = 0.0015 \]

\[ I = I_1 + I_2 = 36 \text{ A} \]

\[ B = \frac{\mu_0}{4\pi} \frac{36(0.0015) \sin(\phi)}{0.08^2} = \frac{36(0.0015)}{0.08^2} \left(\frac{5.2}{2}\right) = 2.74 \times 10^{-7} \text{ T} \]

(Notice that there is a \(\sin()\) factor in the equation. This comes from the cross product)

3) A long, straight wire lies along the \(y\)-axis and carries a current 8.00 A in the \(-y\)-direction. In addition to the magnetic field due to the current in the wire, a uniform magnetic field \(B\) with magnitude \(1.5 \times 10^{-6}\) T is in the \(+x\)-direction. What is the net magnetic field at the following points?: (Book 28.19)

(a) \([0, 0, 1]\) m

\[ B_1 = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 8}{2\pi (1)} = 1.6 \times 10^{-6} \text{ T} \text{ (\(-x\) direction)} \]

\[ B_{\text{tot}} = B_1 - B_0 = 1 \times 10^{-6} \text{ T} \text{ (\(-x\) direction)} \]

(b) \([1, 0, 0]\) m
\[ B_i = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 8}{2\pi (1)} = 1.6(10^{-6}) \, T \quad (+z \, \text{direction}) \]

\[ B_{tor} = \sqrt{B_i^2 + B_0^2} = 2.19(10^{-6}) \, T \quad (\theta = 46.8^\circ \, \text{from} \, +x \, \text{to} \, +z) \]

(c) \([0, 0, -0.25] \, \text{m}\)

\[ B_i = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 8}{2\pi (0.25)} = 6.4(10^{-6}) \, T \quad (+x \, \text{direction}) \]

\[ B_{tor} = B_0 + B_i = 7.9(10^{-6}) \, T \quad (+x \, \text{direction}) \]

4) Two long, parallel wires are separated by a distance of 2.10 cm. The force per unit length that each wire exerts on the other is \(3.70 \times 10^{-5} \, \text{N/m}\), and the wires repel each other. The current in one wire is 0.700 A. What is the magnitude and direction of the current in the second wire? (Book 28.26)

\[ \frac{-F}{L} = \frac{\mu_0 I_1}{2\pi r} \rightarrow I_1 = \frac{F 2\pi r}{L \mu_0} = -10.2 \, \text{A} \]

(Opposite in direction because there is a negative force)

5) Calculate the magnitude of the magnetic field at point \(P\) of the figure in terms of \(I_1, I_2,\) and \(R\). What does your expression give when \(I_1 = I_2\) ? (Book 28.31)

The straight sections of each wire do not create a magnetic field at point \(P\), so the only sections that create a field are the semi-circular portions:

\[ dB = \frac{\mu_0 I_1 dl \times \hat{r}}{4\pi r^2} \rightarrow dB = \frac{\mu_0 I_1 dl \times \hat{r}}{4\pi r^3} \]

\[ dB = \frac{\mu_0 I_1 R}{4\pi R^3} dl = \frac{\mu_0 I_1}{4\pi R^2} dl \]

\[ B_1 = \frac{\mu_0 I_1}{4\pi R^2} \left( \frac{4}{\pi R} \right) \left( \pi R \right) = \frac{\mu_0 I_1}{4 R} \]

\[ B_2 = \frac{\mu_0 I_1}{4 R} \]

\[ B_{net} = B_1 - B_2 = \frac{\mu_0 (I_1 - I_2)}{4R} \]

Thus, when \(I_1 = I_2\), \(B = 0\)
6) A closely wound, circular coil with radius 2.60 cm has 810 turns. (Book 28.32)

(a) What must the current in the coil be if the magnetic field at the center of the coil is 5.60×10⁻² T?

\[ B = \frac{\mu_0 NI}{2a} \rightarrow I = \frac{B(2)r}{\mu_0 N} = 2.86 \text{ A} \]

(b) At what distance x from the center of the coil, on the axis of the coil, is the magnetic field half its value at the center?

\[ \frac{1}{2} \frac{\mu_0 NI}{2r} = \frac{\mu_0 I r^2 N}{2(x^2 + r^2)^{3/2}} \rightarrow x = \sqrt{2^{2/3} r^2 - r^2} = 1.99 \text{ cm} \]